淡江大學八十九學年度日間部轉學生招生考試試題

系別：理工組二年級　　　　科目：微積分

注意：第一題為填充題，請在答案卷第一頁依序寫上題號再寫答案，不必寫出演算過程。

第二、三、四、五、六為計算証明題，務必要有演算過程。

一、填充題（共10小題，每小題5分）

1. Find $\lim_{n\to\infty} \sum_{k=1}^{n} \frac{\pi}{n} \cos\left(\frac{k\pi}{2n}\right)$.

2. Let $f(x) = \begin{cases} x^2 & \text{if } x<1 \\ \alpha x+\beta & \text{if } 1\le x<2 \\ x & \text{if } x\ge 2 \end{cases}$. Determine α and β so that $f(x)$ is continuous everywhere.

3. Find y'' at $(1,1)$ if $xy+y^3=2$.

4. Find the maximum value of $f(x,y)=x^2-y^2+4$ on the set $S=\{(x,y): x^2+y^2\le 1\}$.

5. Find $\int \frac{\sqrt{4-x^2}}{x^2}\,dx$.

6. Let $f(0)=0$ and $f'(0)=2$. Find the derivative of $f(f(f(f(x))))$ at $x=0$.

7. Find the area of region between the parabola $y^2=4x$ and the line $4x-3y=4$.

8. Suppose f is continuous and strictly increasing on $[0,1]$ with $f(0)=0$ and $f(1)=1$. If $\int_0^1 f(x)\,dx=\frac{2}{5}$, calculate $\int_0^1 f^{-1}(y)\,dy$.

9. Find $\lim_{x\to 0^+} x^{(x^x)}$.

10. Find the volume of the solid bounded by the cylinders $x^2=y$ and $z^2=y$, and the plane $y=1$.

《注意背面尚有試題》

本試題雙面印製

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二、(10分) Prove that if f is continuous on $[0,1]$ and satisfies $0 \le f(x) \le 1$ there, then f has a fixed point — that is, there is a number c in $[0,1]$ such that $f(c) = c$.

三、(10分) Evaluate $\int_0^1 \int_0^{\sqrt{1-x^2}} \sin(x^2+y^2)\, dy\, dx$.

四、(10分) Show that the p-series $\sum_{k=1}^{\infty} \frac{1}{k^p}$ converges if $p>1$ and diverges if $p \le 1$.

五、(10分) The point $P(1,-1,-10)$ is on the surface $z = -10\sqrt{|xy|}$. Starting at P, in what direction $u = u_1 i + u_2 j$ should one move in each case?

(a) To climb most rapidly.

(b) To stay at the same level.

六、(10分) Evaluate $\iiint_S 2xyz\, dV$, where the solid region S that is bounded by the parabolic cylinder $z = 2 - \frac{x^2}{2}$ and the planes $z=0$, $y=x$, and $y=0$.