淡江大學八十七學年度日間部轉學生入學考試試題

系別：理工組二年級　　科目：微積分

本試題共 2 頁

注意：第一題為填充題請在答案卷第一頁依序寫上小題號再寫答案，不必寫出演算過程。

第二、三、四、五、六為計算或証明題，必須寫出演算過程

一、填充題（共10小題，每小題5分）

1. Find the equation of the normal line to the surface $x^2+y^2+2z^2=23$ at $(1,2,3)$.

2. Find the local minimum of $f(x,y)=3x^3+y^2-9x+4y$.

3. Find $\int \frac{3x-1}{x^2-x-6}\,dx$.

4. Find the power series for $e^{\tan^{-1}x}$ through terms of degree 4.

5. Find $\lim_{x\to 0} \frac{\int_0^x \sqrt{1+\sin 2t}\,dt}{x}$.

6. Find the area of the region outside the cardioid $r=1+\cos\theta$ and inside the circle $r=\sqrt{3}\sin\theta$.

7. Evaluate $\int_0^{\frac{\pi}{2}} \sin^{10}x\,dx$.（答案可以不必化簡）

8. Calculate $\int_2^4 \frac{\sqrt{x^2-4}}{x}\,dx$.

9. Evaluate $\int_0^4 \int_{\frac{x}{2}}^2 e^{y^2}\,dy\,dx$.

10. Find the volume of the solid S bounded above by $z=4-x^2-y^2$, below by $z=0$, and laterally by $y=0$ and $x^2+y^2=2x$.

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"以下每題10分"

二、Sketch the graph $f(x)=\dfrac{x^2-2x+4}{x-2}$.

三、Prove: Let f have a derivative on $[a,b]$. If $f(a)f(b)<0$ and if $f'(x)\neq 0$ for all x in (a,b), then the equation $f(x)=0$ has one and only one solution between a and b.

四、Test for convergence or divergence:

(1) $\sum_{n=1}^{\infty}\dfrac{n!}{n^n}$　　(2) $\sum_{n=2}^{\infty}\dfrac{1}{n\ln n}$.

五、Show that $\int_0^{\infty}e^{-x^2}dx=\dfrac{\sqrt{\pi}}{2}$.

六、Find the volume of the solid inside both of the spheres $\rho=2\sqrt{2}\cos\phi$ and $\rho=2$. (Use spherical coordinates)