淡江大學八十七學年度碩士班入學考試試題

系別： 航空太空工程學系　　　　科目： 動力學

本試題共 乙 頁

1. (20%) Refer to Figure 1, a particle of mass m is attached to a massless string of length $2R$. As the deflection angle θ increases, the string wraps around a fixed cylinder of radius R next to the support point O. Find: (a) the differential equation for θ, where θ is positive; (b) the maximum value of θ, assuming
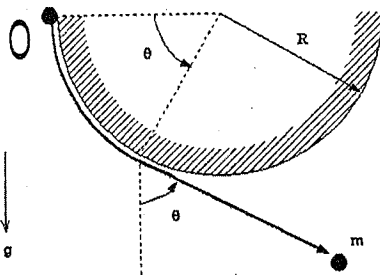$$\theta(0) = 0, \quad \dot{\theta}(0) = \sqrt{g/2R}.$$

2. (20%) Refer to Figure 2, a particle of mass m moves vertically downward with velocity v_0 strikes a smooth triangular block of mass $2m$ which is initially stationary. Assuming a coefficient of restitution e, find the velocities of the block and the particle immediately after impact.
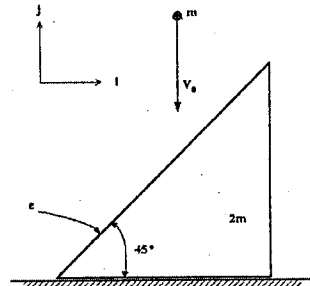

Figure 1

Figure 2

3. (30%) Refer to Figure 3, a particle of mass m moves on the smooth inner surface of a fixed inverted circular cone having a vertex angle 2α.

    (a) Using the angle ϕ about the vertical axis of symmetry and the distance r measured from the vertex as generalized coordinates, obtain the differential equations of motion.

    (b) Find $\dot{r}_{max}$, assuming $\alpha = 30°$ and initial conditions $r(0) = r_0$, $\dot{r}(0) = 0$, $\dot{\phi}(0) = 4\sqrt{g/r_0}$.

    (c) Now suppose the particle moves in a steady circular motion on the cone and $r = r_0$. Write the differential equation for small perturbations in r and solve for circular frequency ω of this motion

4. (30%) An axially-symmetric satellite with gravity gradient stabilization is in a circular orbit about the earth. Using the notation of Figure 4, and assuming $I_t = 10 I_a$ and $\Omega = 2\omega_0$, where ω_0 is the orbital angular velocity and the total spin Ω is about the positive x-axis, solve for the natural frequencies and amplitude ratios (A_θ / A_ψ) of small oscillations of the symmetry axis near its equilibrium position. Find the average value of θ.
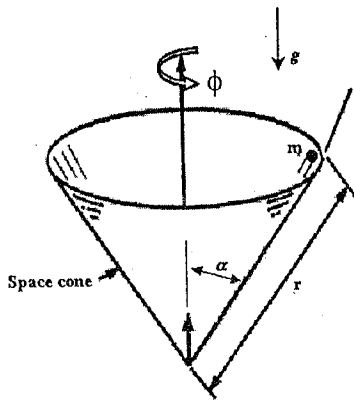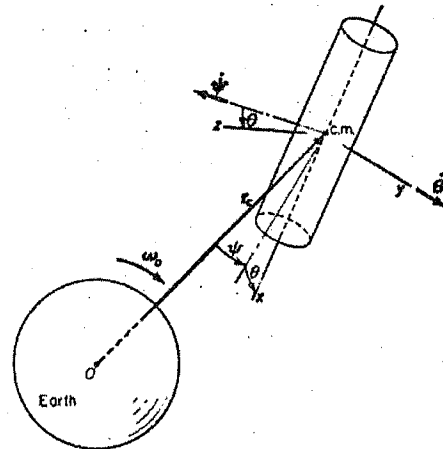

Figure 3

Figure 4