1. (25 points) A real matrix is given as

$$\mathbf{A} = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 0 \\ -3 & 3 & 5 \end{bmatrix}$$

   (a) (10 points) Determine all eigenvalues (regular eigenvectors, and generalized eigenvectors) of the matrix.

   (b) (15 points) Transform the matrix into its Jordan form by constructing modal matrix with the regular and generalized eigenvectors.

2. (10 points) Compute e^{At} for the following matrix:

$$\mathbf{A} = \begin{bmatrix} -3 & 1 \\ 0 & -2 \end{bmatrix}$$

3. (25 points) Consider the initial value problem:

$$\mathbf{x}' = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 2 & 0 \\ 1 & 0 & 3 \end{bmatrix} \mathbf{x}, \quad \mathbf{x}_0 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}.$$

   (a) (5 points) Find the eigenvalues.

   (b) (10 points) Obtain the associated eigenvectors.

   (c) (10 points) Obtain the solution of the initial value problem.

4. (20 points) Find the inverse Laplace Transforms of the functions:

   (a) (10 points) $\dfrac{s+2}{s^2+4s+13}$

   (b) (10 points) $\dfrac{1}{s(s+1)}$

5. (20 points) Consider two coordinate frames:

   $x_1 x_2 x_3$ _with unit vectors_ $\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3$

   $x_1' x_2' x_3'$ _with unit vectors_ $\mathbf{e}_1', \mathbf{e}_2', \mathbf{e}_3'$

   If their origins coincide, as shown in Fig.P5, the unit vectors $\mathbf{e}_1', \mathbf{e}_2', \mathbf{e}_3'$ can be expressed in terms of the unit vectors $\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3$ as

$$\begin{aligned}
\mathbf{e}_1' &= (\mathbf{e}_1' \cdot \mathbf{e}_1)\mathbf{e}_1 + (\mathbf{e}_1' \cdot \mathbf{e}_2)\mathbf{e}_2 + (\mathbf{e}_1' \cdot \mathbf{e}_3)\mathbf{e}_3 \\
\mathbf{e}_2' &= (\mathbf{e}_2' \cdot \mathbf{e}_1)\mathbf{e}_1 + (\mathbf{e}_2' \cdot \mathbf{e}_2)\mathbf{e}_2 + (\mathbf{e}_2' \cdot \mathbf{e}_3)\mathbf{e}_3 \\
\mathbf{e}_3' &= (\mathbf{e}_3' \cdot \mathbf{e}_1)\mathbf{e}_1 + (\mathbf{e}_3' \cdot \mathbf{e}_2)\mathbf{e}_2 + (\mathbf{e}_3' \cdot \mathbf{e}_3)\mathbf{e}_3
\end{aligned} \qquad (5\text{-}1)$$

   Let the frame $x_1 x_2 x_3$ rotates about the x_2 axis with an angle of θ to obtain the frame $x_1' x_2' x_3'$.

(a) (10 points) Referring to the Fig. P5 and Eq. (5-1), obtain the transformation matrix R from frame $x_1 x_2 x_3$ to frame $x_1' x_2' x_3'$.

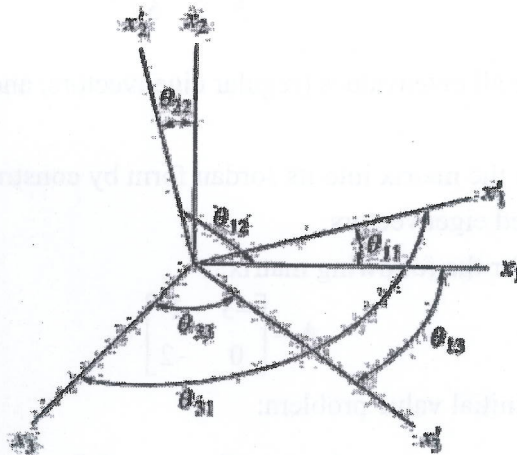(b) (10 points) Show that the transformation matrix R is an orthogonal matrix.

Figure P5.