淡江大學 102 學年度碩士班招生考試試題

系別: 航空太空工程學系　　科目: 工程數學

考試日期: 3月10日 (星期日) 第2節　　本試題共 6 大題, 1 頁

1. Determine if each of the following set of vectors is linearly independent or dependent? Give reasons for your answers.
   (a) (5 points) $(1,-1,1),(1,1,-1),(-1,1,1)$, and $(0,1,0)$.
   (b) (5 points) $(1,2,3),(2,3,0),(1,0,0)$.

2. Consider the following system of equations:

$$\begin{aligned} x_1 + 4x_2 - 2x_3 + 8x_4 &= 12, \\ x_2 - 7x_3 + 2x_4 &= -4, \\ 3x_3 + 9x_4 &= -15, \\ x_3 + 3x_4 &= -5. \end{aligned} \tag{1}$$

   Apply the fundamental theorem of linear systems and Gauss elimination to determine
   (a) (5 points) Does a solution exist? Why?
   (b) (5 points) Is the solution unique? Why?
   (c) (5 points) Obtain all solution(s).

3. Given the function $f(x,y,z) = x^2y - 2xy + 4yz$ and the point $P:(1,2,-1)$.
   (a) (10 points) Find the directional derivative of f in the direction $a = (0,-2,0)$ at the point P.
   (b) (10 points) Determine the direction of maximum increase in the function f at the point P.

4. Let the matrix $D = P^{-1}AP$, where

$$D = \begin{bmatrix} 6 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & -3 \end{bmatrix} \text{ and } P = \begin{bmatrix} 2 & 0 & 3 \\ 0 & 1 & 0 \\ 3 & 0 & 5 \end{bmatrix}. \tag{2}$$

   (a) (10 points) What are the eigenvalues of the matrix A^3.
   (b) (10 points) Find all the correspond eigenvectors of the matrix A^3.

5. (15 points) Use TWO different methods to solve the following first-order initial value problem

$$\frac{d}{dx}y = y - x, y(0) = \frac{2}{3}. \tag{3}$$

6. In an oil refinery, a storage tank contains 2000 gal of gasoline that initially has 100 lb of an additive dissolved in it. In preparation for winter weather, gasoline containing 2 lb of additive per gallon is pumped into the tank at a rate of 40 gal/min. The well-mixed solution is pumped out at a rate of 45 gal/min (See Figure 1).
   (a) (10 points) Write down the differential equation that describes the process. Let y be the amount (in pounds) of additive in the tank, and V be the number of gallons of gasoline and additive in solution in the tank at time t (min), respectively.
   (b) (10 points) How much of the additive is in the tank 20 min after the pumping process begins?

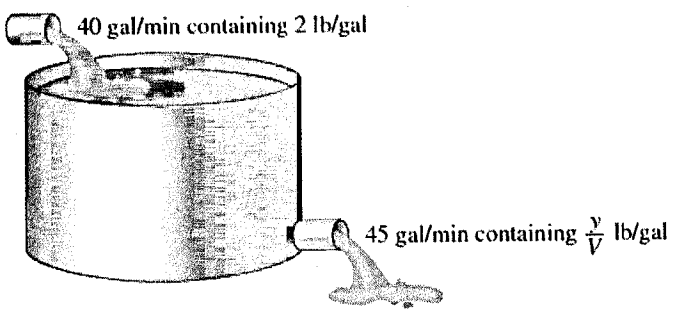

Figure 1: The storage tank in Example 6 mixes input liquid with stored liquid to produce an output liquid

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