

# 淡江大學 101 學年度碩士班招生考試試題

系別：航空太空工程學系

科目：工 程 數 學

考試日期：2 月 26 日(星期日) 第 3 節

本試題共 五 大題，乙 頁

1. (20 points) A real, symmetric matrix  $\mathbf{A}$  is given as

$$\mathbf{A} = \begin{bmatrix} 2 & 1 & 0 \\ 1 & -2 & 4 \\ 0 & 4 & 2 \end{bmatrix}$$

- (a) Find the eigenvalues and associated eigenvectors,  
(b) Find a real orthogonal matrix that diagonalizes  $\mathbf{A}$ .

2. (20 points) Find all solutions of  $\mathbf{x}' = \mathbf{A}\mathbf{x}$ , where the matrix  $\mathbf{A}$  is as given below.

(a)  $\mathbf{A} = \begin{bmatrix} 4 & 2 \\ 3 & 3 \end{bmatrix}$ .

(b)  $\mathbf{A} = \begin{bmatrix} 5 & -4 & 4 \\ 12 & -11 & 12 \\ 4 & -4 & 5 \end{bmatrix}$

3. (20 points) Consider the initial value problem:

$$\mathbf{x}' = \begin{bmatrix} -2 & -1 & -5 \\ 25 & -7 & 0 \\ 0 & 1 & 3 \end{bmatrix} \mathbf{x}, \quad \mathbf{x}_0 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}.$$

- (a) Find the eigenvalues.  
(b) Obtain the associated eigenvectors.  
(c) Obtain the solution of the initial value problem.

4. (20 points) Find the inverse Laplace transform of each function.

(a)  $\frac{4}{(s^2 + 4s + 20)}$ .

(b)  $\frac{1}{s(s-4)^2}$ .

5. (20 points) Use Laplace transform to solve the following initial value problem.

$$\begin{aligned} x'' - 2x' + 3y' + 2y &= 4, \\ 2y' - x' + 3y &= 0, \\ x(0) = x'(0) = y(0) &= 0. \end{aligned}$$