淡江大學99學年度碩士班招生考試試題

系別：航空太空工程學系　　科目：工 程 數 學

准帶項目請打「V」	
V	計 算 機

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1. (20 points) Find the general solutions (real-valued) of the systems below.

   (a) $x_1' = 3x_1 + 5x_2;\ \ x_2' = -x_1 + x_2.$

   (b) $x_1' = 2x_1 - x_2,\ \ x_2' = 6x_1 + 2x_2.$

2. (20 points) Find all solutions of $\mathbf{x}' = \mathbf{A}\mathbf{x}$, where the matrix $\mathbf{A}$ is as given below.

   (a) $\mathbf{A} = \begin{bmatrix} 5 & 3 \\ -1 & 1 \end{bmatrix}.$

   (b) $\mathbf{A} = \begin{bmatrix} 2 & -1 \\ 8 & -2 \end{bmatrix}.$

3. (20 points) Consider the initial value problem:

$$\mathbf{x}' = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 2 & 0 \\ 1 & 0 & 3 \end{bmatrix}\mathbf{x},\ \ \mathbf{x}_0 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}.$$

   (a) Find the eigenvalues.

   (b) Obtain the associated eigenvectors.

   (c) Obtain the solution of the initial value problem.

4. (20 points) Find the inverse Lapalace transform of each function.

   (a) $\dfrac{1}{s^2(s-a)}.$

   (b) $\dfrac{1}{s(s^2+1)}.$

5. (20 points) Use Laplace transform to solve the following initial value problem.

$$y'' + 4y' + 20y = e^{-3t},\ \ y(0) = -1,\ \ y'(0) = 2.$$