

淡江大學 97 學年度碩士班招生考試試題

79

系別：航空太空工程學系

科目：工 程 數 學

准帶項目請打「V」	
✓	簡單型計算機

本試題共 乙 頁，五 大題

1. (20 points) Find the general solutions (real-valued) of the systems below.

(a) $x_1' = 5x_1 + 3x_2; x_2' = -x_1 + x_2.$

(b) $x_1' = 2x_1 - x_2, x_2' = 8x_1 - 2x_2.$

2. (20 points) Find all solutions of $\mathbf{x}' = \mathbf{A}\mathbf{x}$, where the matrix \mathbf{A} is as given below.

(a) $\mathbf{A} = \begin{bmatrix} 3 & -2 \\ 2 & -2 \end{bmatrix}.$

(b) $\mathbf{A} = \begin{bmatrix} 3 & -2 \\ 2 & -1 \end{bmatrix}.$

3. (20 points) Consider the initial value problem:

$$\mathbf{x}' = \begin{bmatrix} -1 & 1 & 0 \\ -1 & 0 & 1 \\ 1 & 0 & -2 \end{bmatrix} \mathbf{x}, \quad \mathbf{x}_0 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}.$$

(a) If one of its eigenvalues is -1 , find the other two eigenvalues.

(b) Obtain the associated eigenvectors.

(c) Obtain the solution of the initial value problem.

4. (20 points) Use Convolution Theorem to find the inverse Laplace transform of each function. Evaluate the integral to your limits.

(a) $\frac{s}{(s^2 + 1)^2}.$

(b) $\frac{\mathcal{L}[f]}{s^2 + 1}.$

5. (20 points) Use Laplace transform to solve the following initial value problem.

$$y'' - 7y' + 12y = 6e^{3t}, \quad y(0) = -1, \quad y'(0) = 2.$$