1. Solve $y'' + y' = te^t$; $y(0) = 0$, $y'(0) = 0$.   (20%)

2. Use the Convolution theorem to find the inverse Laplace transform of each function

    (a) $\dfrac{s}{(s+1)(s+2)^3}$,   (10%)

    (b) $\dfrac{s^2 + 4s + 4}{(s^2 + 4s + 13)^2}$.   (10%)

3. Find the general solution of

    $$\mathbf{x}' = \begin{bmatrix} -8 & -1 \\ 16 & 0 \end{bmatrix} \mathbf{x}.$$ (20%)

4. Let **A** be *unitary*, *hermitian*, or *skew-hermitian*. Prove that

    $$\overline{\mathbf{A}}\mathbf{A}^T = \overline{\mathbf{A}}\mathbf{A}.$$ (20%)

5. Use the Method of Separation of variables to construct a series solution of the heat equation

    $$\frac{\partial u}{\partial t} - k\frac{\partial^2 u}{\partial x^2} = 0,\ \ 0 < x < L,\ \ t > 0,\ \ k \text{ is a constant,}$$

    and the following initial and boundary conditions

    $$u(0,t) = u(L,t) = 0,$$
    $$u(x,0) = \sin(2\pi x / L).$$ (20%)