淡江大學九十四學年度碩士班招生考試試題 ㄐㄐ-1

系別：航空太空工程學系　　　科目：工 程 數 學

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1. Solve $y'' + 5y' + 6y = 3\delta(t-2) - 4\delta(t-5)$; $\quad y(0) = y'(0) = 0$. $\quad$ (20%)

   Note: Here $\delta(\ )$ is the Dirac delta function.

2. Find the family of orthogonal trajectories of the family $y^2 = Kx^3$, where K is a

   constant. $\quad$ (20%)

   Note: Sometimes we encounter two families of curves in which any curve of one

   family is orthogonal to any curve of the other wherever two such curves

   intersect. Two families of curves that are in this sense are called orthogonal

   trajectories.

3. Solve $x^2 y'' - 2xy' + 2y = 10\sin(\ln(x))$; $\quad y(1) = 3, \ y'(1) = 0$. $\quad$ (20%)

4. Find the inverse of the following matrix or else show that this matrix is singular.

$$A = \begin{bmatrix} -2 & 6 & 0 & 0 \\ 1 & 4 & 4 & 11 \\ 4 & -4 & -5 & 3 \\ -3 & 1 & 2 & -6 \end{bmatrix}. \qquad (10\%)$$

5. Find the Fourier series of $f(x) = x^4$ on $[-\pi, \pi]$. $\quad$ (15%)

6. Use the Laplace transform to solve the following boundary value problem

$$a^2 \frac{\partial^2 y}{\partial x^2} = \frac{\partial^2 y}{\partial t^2}, \qquad (x > 0, \ t > 0; \ a \text{ is a constant.})$$

$$y(0,t) = t, \qquad (t > 0)$$

$$\frac{\partial y}{\partial t}(x,0) = B, \qquad (x > 0; \ B \text{ is a constant.})$$

$$y(x,0) = 0. \qquad (x > 0) \qquad (15\%)$$