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淡江大學八十九學年度碩士班招生考試試題

系別：航空太空工程學系　　科目：工程數學

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1. Evaluate the integral $\int_C \exp(\frac{1}{z^2})dz$, where C is the circle $|z|=2$ described in the positive sense. (15%)

2. Find the volume integral of $f(x,y,z)=x+yz$ over the box bounded by the coordinate planes, $x=1, y=2$, and $z=1+x$. (15%)

3. It is known that the Gamma function $\Gamma(\frac{1}{2})=\sqrt{\pi}$. Please evaluate the value of $\int_0^\infty \sqrt{ze^{-z^3}}\,dz$. (15%)

4. A differential equation is written in a 2X2 matrix form as $\{x''\}_{2X1}+[K]_{2X2}\{x\}_{2X1}=0$, the $[K]$ matrix is symmetric. Prove that the eigen-values of this 2^{nd} order *O.D.E.* system are real numbers. (25%)

5. An elastically-pinned beam is shown in the figure. The beam bending equation of motion is sown below,

$$\frac{\partial^2 y}{\partial t^2}+a^4\frac{\partial^4 y}{\partial x^4}=0, \quad \text{where } a^4=\frac{EI}{\rho A}=\text{beam property}=\text{constant}.$$

The boundary conditions are,

$$X''(0)=X'''(0)=0, \qquad X(l)=0,\ EIX''(l)=-KX'(l), \text{ where } EI \text{ and } K \text{ are constant.}$$

The spatially dependent portion of the general solution for this beam bending dynamics will be used as,

$$X(x)=D1(\sin\alpha x+\sinh\alpha x)+D2(\sin\alpha x-\sinh\alpha x)+D3(\cos\alpha x+\cosh\alpha x)+D4(\cos\alpha x-\cosh\alpha x)$$

Please find the coefficients *D2*, *D4*, and propose a way (but **do not solve**) to find the coefficients *D1*, *D3*, the eigen-value α, and express the vibration frequency of the beam as a function of α, EI, ρ ,and A. (30%)