淡江大學八十八學年度碩士班招生考試試題

系別：航空及太空工程學系　　科目：工程數學　　本試題共 1 頁

1. (a) Let A be an $n\times n$ matrix with eigenvalues λ_1, λ_2, λ_3,..., λ_n and suppose that P diagonalizes A. Prove that for any positive integer k,

$$A^k = P\begin{bmatrix} \lambda_1^k & 0 & 0 & \cdots & 0 \\ 0 & \lambda_2^k & 0 & \cdots & 0 \\ 0 & 0 & \lambda_3^k & 0 & \vdots \\ \vdots & \vdots & 0 & \ddots & 0 \\ 0 & 0 & \cdots & 0 & \lambda_n^k \end{bmatrix} P^{-1}. \quad (15\%)$$

(b) Computer A^{50} if $A=\begin{bmatrix} -1 & 0 \\ 1 & -5 \end{bmatrix}$. (10%)

2. If $\begin{cases} \dfrac{dx(t)}{dt} + 2x(t) + 3y(t) = 1 \\ \dfrac{dy(t)}{dt} + 3x(t) + 2y(t) = 0 \end{cases}$, find $\lim\limits_{t\to\infty} x(t)$ and $\lim\limits_{t\to\infty} y(t)$. (15%)

3. Evaluate the integral

$$\oint_\Gamma \frac{e^z(z^2-4)^2}{(z+i)^2}dz,$$

where Γ is the counterclockwise circle $|z-1+2i|=4$. (10%)

4. Suppose an elastic string is fixed at $x=0$ and extends from 0 to ∞. A boundary value problem for the motion of the string is given by

$$\frac{\partial^2 y}{\partial t^2} = a^2\frac{\partial^2 y}{\partial x^2} \qquad (x>0, t>0),$$

$$y(0,t)=0 \qquad (t>0),$$

$$y(x,0)=f(x), \quad \frac{\partial y}{\partial t}(x,0)=0 \qquad (x>0),$$

where $f(x)=\begin{cases} x, & 0\le x\le 1, \\ 1, & 1\le x\le 4, \\ 5-x, & 4\le x\le 5, \\ 0, & x\ge 5. \end{cases}$

Find the solution of $y(x,t)$. (25%)

5. Solve the following differential equations

(a) $y''+2y'+y=\dfrac{e^{-x}}{x}$. (15%)

(b) $y''+5y'+4\delta(t-5)=3\delta(t-2)-6y$; $y(0)=y'(0)=0$, where $\delta(\,)$ is the Dirac delta function. (10%)