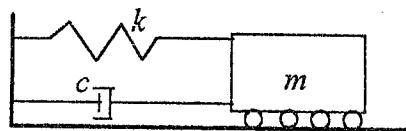# 淡江大學八十七學年度碩士班入學考試試題

系別： 航空太空工程學系　　　　科目： 工程數學

1. Given the equation of motion of a mass-spring-damper system as follows,
$$m\ddot{x} + c\dot{x} + kx = 0$$
suppose that the initial displacement and velocity are all positive, make a neat plot of $x(t)$ *vs.* t **and** explain the motion of this system if
   (a) $c^2 > 4mk$. (5%)
   (b) $c^2 < 4mk$. (5%)
   (c) $c^2 = 4mk$. (5%)
   (d) $c = 0$.　　(5%)

2. Compute the line integral $\int \vec{F} \, d\vec{R}$ from $(0,0,0)$ to $(1,2,4)$ if

   $$\vec{F} = x^2 \vec{i} + y\vec{j} + (xz - y)\vec{k}$$
   (a) along the line segment joining these two points. (5%)
   (b) along the curve given parametrically by $x = t^2, y = 2t, z = 4t^3$. (5%)

3. Given an O.D.E. as follows,
   $(1 - x^2)y'' - 2xy' + 2y = 0 \quad (0 < x < 1)$
   with the boundary conditions $y(0) = 1, y'(0) = 0$.
   Find the solution of y. (20%)

4. A string with length L is fixed at both ends. The string vibration equation of motion is as follows, $\frac{\partial^2 y}{\partial t^2} = a^2 \frac{\partial^2 y}{\partial x^2}, \quad 0 < x < L, \quad t > 0$, the initial conditions are given as $y(x,0) = f(x)$, and $\dot{y}(x,0) = 0$. Find
   (a) the solution of $y(x,t)$. (15%)
   (b) frequency of this string (5%)
   (c) plot the first three modes of this string. (10%)

5. Find a function which satisfies the differential equation

   $$\frac{d^2 f(t)}{dt^2} + 2\frac{d f(t)}{dt} + 2f(t) = \left\langle \begin{array}{ll} \sin t, & |t| \le 6\pi, \\ \\ 0, & otherwise. \end{array} \right\rangle \qquad (20\%)$$