

淡江大學 99 學年度碩士班招生考試試題

系別：機械與機電工程學系

科目：工程數學

准帶項目請打「V」

計算機

本試題共 一頁，七 大題

- 1.(10%) Figure 1 shows a particle P moving around a unit circle with an angular velocity $\dot{\theta}(t)$ and an angular acceleration $\ddot{\theta}(t)$. The position vector \mathbf{r} of this particle is

$$\mathbf{r} = \cos\theta(t)\mathbf{i} + \sin\theta(t)\mathbf{j}$$

where t is time measured in seconds, \mathbf{i} and \mathbf{j} are unit vectors along the fixed x and y axes, respectively.

- a) Obtain the velocity vector $\dot{\mathbf{r}}$ of this particle.
- b) Obtain the acceleration vector $\ddot{\mathbf{r}}$ of this particle.

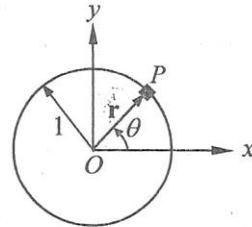


Figure 1

- 2.(15%) The acceleration function $a(t)$ of a particle moving along a straight line is given by

$$a(t) = \begin{cases} \frac{t^2}{2}; & 0 \leq t \leq 1 \\ \frac{1}{2t^2}; & t \geq 1 \end{cases}$$

where t is time measured in seconds.

- a) Obtain the velocity function $v(t)$ of this particle if the initial velocity is zero and $v(t)$ is continuous at $t = 1$.
- b) Obtain the displacement function $s(t)$ of this particle if it starts from the initial position $s = 0$ and $s(t)$ is continuous at $t = 1$.

- 3.(15%) What is the condition for the three constants a , b , and c so that the following system of equations has solutions for x , y , and z ? explain your answer.

$$2x - 3y + 4z = a$$

$$-x + y - 2z = b$$

$$x - 2y + 2z = c$$

- 4.(15%) Determine eigenvalues (i.e. characteristic values) and eigenvectors of matrix A given below.

$$A = \begin{bmatrix} 3 & 2 \\ -2 & 7 \end{bmatrix}$$

- 5.(15%) A function $y(x)$ satisfies the equation

$$y(x) \frac{dy(x)}{dx} = e^x; \quad x \geq 0 \tag{1}$$

- a) Is equation (1) a linear equation? Why?
 - b) Obtain $y(x)$ if $y(0)=1$.
- 6.(15%) Determine the function $u(x)$ which satisfies the equation

$$\frac{d^2u(x)}{dx^2} + k^2u(x) = x; \quad 0 \leq x \leq L$$

and the conditions $u(0) = u(L) = 0$. k is a real constant and $k \neq 0$.

- 7.(15%) The function f is defined by $f(x, y) = 2x^2 \ln(xy^2)$, where \ln is the natural logarithm function. Determine the change rate of f at $(x, y) = (1, 1)$ in the direction $3\mathbf{i} - 4\mathbf{j}$.