

淡江大學八十八學年度碩士班招生考試試題

系別：機械工程學系

科目：工程數學

本試題共 1 頁

1. Solve

$$y' - 4y = H(t)e^{-4t} \quad (10\%)$$

for all real t .

2. Find the general solution of

$$y'' - \frac{4}{x}y' + \frac{4}{x^2}y = x^2 + 1. \quad (15\%)$$

3. Solve the initial value problem

$$y'' + 2ty' - 4y = 1; y(0) = y'(0) = 0. \quad (15\%)$$

4. Solve

$$\oint_C \frac{-y}{x^2+y^2} dx + \frac{x}{x^2+y^2} dy \quad (10\%)$$

with C any simple closed path lying in the x, y -plane but not passing through the origin.

5. Let $\mathbf{F} = -y\mathbf{i} + x\mathbf{j} - xyz\mathbf{k}$ and consider the surface Σ consisting of the part of the cone $z = \sqrt{x^2 + y^2}$ for $x^2 + y^2 \leq 9$. Compute both side of the conclusion of Stokes's theorem. (15%)

6. Let

$$f(x) = \begin{cases} 1, & 0 \leq x \leq \frac{\pi}{2}, \\ 2, & \frac{\pi}{2} < x \leq \pi. \end{cases} \quad (15\%)$$

Expand this function in Fourier sine and cosine series on $[0, \pi]$.

7. Solve the partial differential equation

$$\begin{aligned} \frac{\partial u}{\partial t} &= a^2 \frac{\partial^2 u}{\partial x^2} \quad (0 < x < L, t > 0), \\ u(0, t) &= 0, \quad \frac{\partial u}{\partial x}(L, t) = -Au(L, t) \quad (t \geq 0), \\ u(x, 0) &= f(x) \quad (0 \leq x \leq L). \end{aligned} \quad (20\%)$$