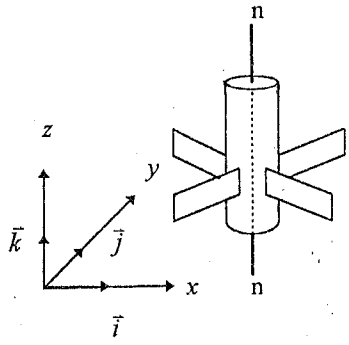
淡江大學八十七學年度碩士班入學考試試題

系別： 機械工程學系　　科目： 工程數學

說明：一共有四題，每題都必須回答；可用中文回答。

1.(25%)

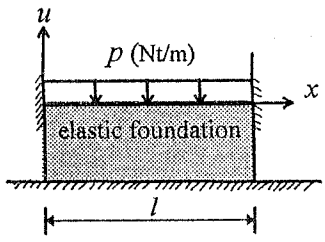


The device shown in the figure may rotate about the n-n axis.

(a) A two dimensional fluid flow is produced by a source at the origin. In this case the velocity is given by $\vec{v} = f(r)\vec{u}_r$ where f is a function of $r = \sqrt{x^2 + y^2}$, and $\vec{u}_r = (x\vec{i} + y\vec{j})/r$ is the unit vector in the direction of r. Does this flow bring the device into rotation? Prove your answer.

(b). The velocity function of a flow in the y direction is given by $\vec{v} = v_0 \ln\left(\frac{a-s}{a+s}\right)\vec{j}$, where v_0 and a are constants, and $s = \sqrt{a^2 - x^2}$. Does this flow bring the device into rotation? Prove your answer.

2.(25%)



A string resting on an elastic foundation is subjected to a uniform load p. The differential equation for the displacement u of this string is

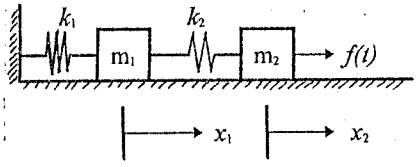
$$\frac{d^2u}{dx^2} - ku = p$$

where k is modulus of the elastic foundation. If the string is fixed at both ends, *i.e.* if $u(0)=u(l)=0$,

(a). determine the displacement curve $u(x)$.

(b). what is the displacement curve $u(x)$ when $k \to \infty$?

3.(25%)



The mechanical system shown in the figure consists of two particles of masses m_1 and m_2, which are connected to two linear springs with spring constants k_1 and k_2 respectively. Equations of motion for this system are

$$m_1\ddot{x}_1 + k_1x_1 - k_2(x_2 - x_1) = 0$$
$$m_2\ddot{x}_2 + k_2(x_2 - x_1) = f(t)$$

It is known that $m_1 = m_2 = m$, $k_1 = 3k$, $k_2 = 2k$, and the system is vibrating under no external forces (*i.e.* $f(t)=0$).

(a). By assuming $x_1 = A\ sin(\omega t+\phi)$, and $x_2 = B\ sin(\omega t+\phi)$, obtain two natural frequencies of this system as functions of $z = k/m$.

(b). Obtain two natural modes (eigenvectors) of this system.

(c). Obtain solutions $x_1(t)$ and $x_2(t)$ for free vibrations ($f(t)=0$).

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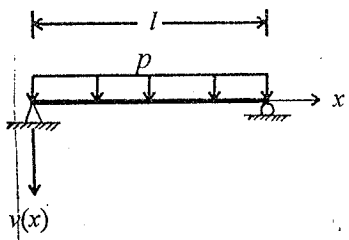
4.(25%)



A simply supported beam is subjected a uniform load p. The deflection curve $v(x)$ satisfies the differential equation

$$\frac{d^4v}{dx^4} = \frac{p}{EI}, \qquad (1)$$

and the boundary conditions $v(0) = v(l) = 0$. The constant quantity EI is called flexural rigidity of the beam.

This problem is to be solved by using **Fourier series**.

(a).Expand the function $f(x) = p/EI$ (= constant) into a **Fourier sine series** in the range $0 \leqq x \leqq l$.

(b).Let $v(x) = \sum_{n=1}^{\infty} a_n \sin\frac{n\pi x}{l}$, determine a_n by substituting this expression into equation (1) and using results in (a).

(c).Is it good to solve this problem by using the Fourier cosine series ? Why ?