

系別：機械與機電工程學系

科目：工 程 數 學

准帶項目請打「V」

簡單型計算機

本試題共 壹 頁，五 大題

1. (20%) Solve $2(y^3 - 2) + 3xy^2y' = 0$; $y(3) = 1$.

2. (20%) Solve $y'' + y' - 6y = 50xe^{2x}$.

3. (15%) Evaluate the line integral $\int_C \vec{F} \cdot d\vec{R}$, where

$$\vec{F} = \cos(x)\vec{i} - y\vec{j} + xz\vec{k} \text{ and } \vec{R} = t\vec{i} - t^2\vec{j} + \vec{k} \text{ for } 0 \leq t \leq 3.$$

4. (20%) Find an orthogonal matrix that diagonalizes the matrix A .

$$A = \begin{bmatrix} 0 & 0 & \sqrt{2} \\ 0 & -1 & 0 \\ \sqrt{2} & 0 & 1 \end{bmatrix}.$$

5. (25%) Solve the partial differential problem.

$$\frac{\partial u}{\partial t} = 9 \frac{\partial^2 u}{\partial x^2} \text{ for } 0 < x < \infty, t > 0,$$

$$u(0, t) = 0 \text{ for } t \geq 0,$$

$$u(x, 0) = \begin{cases} \pi - x & \text{for } 0 \leq x \leq \pi \\ 0 & \text{for } x > \pi \end{cases}$$