系別：化學工程學系　　　科目：工　程　數　學

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本試題共 壹 頁

1. Find the Fourier series expansion of the following function with a period equal to 4.

$$f(x) = 1 \quad (-1 < x < 1), \quad f(x) = 0 \ (1 < x < 3).$$ (20%)

Given:

The Fourier series of a function f defined on the interval $(-p, p)$ is given by

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos \frac{n\pi}{p} x + b_n \sin \frac{n\pi}{p} x \right).$$

where

$$a_0 = \frac{1}{p} \int_{-p}^{p} f(x) \, dx$$

$$a_n = \frac{1}{p} \int_{-p}^{p} f(x) \cos \frac{n\pi}{p} x \, dx$$

$$b_n = \frac{1}{p} \int_{-p}^{p} f(x) \sin \frac{n\pi}{p} x \, dx.$$

2. Find the eigenvalues and eigenfunctions of the following Sturm-Liouville problem. (20%)

$$y'' + \lambda y = 0, \ y(0) = y(2\pi), \ y'(0) = y'(2\pi)$$

3. Solve the following system of linear algebraic equations: (20%)

$$6x_1 - 2x_2 + 2x_3 + 4x_4 = 16$$
$$12x_1 - 8x_2 + 6x_3 + 10x_4 = 26$$
$$3x_1 - 13x_2 + 9x_3 + 3x_4 = -19$$
$$-6x_1 + 4x_2 + x_3 - 18x_4 = -34$$

4. Solve (20%)

$$y' + \frac{y}{x} = 3x^2 y^3$$ (Hint: Bernoulli equation, set $z(x) = y^{-2}$)

5. Solve (20%)

$$x(x-1)y'' + (3x-1)y' + y = 0$$