淡江大學八十七學年度碩士班入學考試試題

系別： 化學工程學系　　　科目： 工程數學

1. Find the eigenvalues and eigenfunctions of the following Sturm-Liouville problem on $[0, \pi/2]$. (20%)

$$y'' + \lambda y = 0; y(0) = y(\frac{\pi}{2}) = 0$$

2. Solve (20%)

$$y'' - 4xy' + (4x^2 - 2)y = 0$$

3. A cylindrical tank 1.50 meters high stands on its circular base (20%) of diameter 1.00 meter and is initially filled with water. At the bottom of the tank there is a hole of diameter 1.00 cm, which is opened at some instant, so that the water starts draining under the influence of gravity (Fig. 1). Find the height $h(t)$ of the water in the tank at any time t. Find the times at which the tank is one-half full, one-quarter full, and empty.

Physical information. Experiments show that water issues from a hole with velocity

$$v(t) = 0.600\sqrt{2gh(t)}$$

(continued)

where t is time, *h(t)* the instantaneous height of the water above the hole, g = 980 cm/sec^2 = 32.17 ft/sec^2 the acceleration of gravity at the surface of the earth.

4. Find the Fourier series expansion for the following periodic function　　　　(20%)

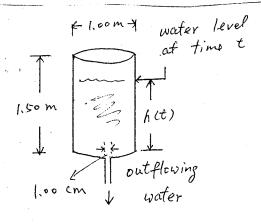$$f(x) = \begin{array}{l} -k \quad \text{if} -\pi < x < 0 \\ k \quad \text{if } 0 < x < \pi \end{array} \quad \text{and } f(x + 2\pi) = f(x)$$

(K is a constant)

5. Find the work done in moving a particle once around a circle C in the xy plane,　(20%)

    if the circle has center at the origin and radius 3 and if the force field is given by

$$\underline{F} = (2x-y+z)\, \underline{i} + (x+y-z^2)\, \underline{j} + (3x-2y+4z)\, \underline{k}$$

(Note: Underlined symbols mean vectorial quantities)

(Fig. 1)