淡江大學九十一學年度碩士班招生考試試題

系別：土木工程學系　　科目：工 程 數 學

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1. (25%)

Given a matrix $A=\begin{bmatrix} 1-p & -1 & 0 \\ 0 & 2-p & 0 \\ 0 & -3 & 3-p \end{bmatrix}$, where p is a constant.

(a) (9%) Determine the relationship between the rank of A and the value of p.

(b) (8%) Let p=0, determine the eigenvalues and eigenvectors.

(c) (8%) Let p=4, determine the inverse, A^{-1}.

2. (25%)

Solve the following linear system of differential equations by the Laplace transform.

$$\begin{cases} y_1''-y_2'=2\cos 2t \\ y_2''+y_1'=-2\sin 2t \end{cases}, \text{ where } y_1'=\frac{dy_1}{dt}, y_2'=\frac{dy_2}{dt}, y_1''=\frac{d^2y_1}{dt^2}, y_2''=\frac{d^2y_2}{dt^2}$$

$$y_1(0)=-1, y_2(0)=1, y_1'(0)=1, y_2'(0)=2$$

3. (25%)

Given the equation $a\frac{d^2y}{dx^2}+b\frac{dy}{dx}+cy=r(x)$, where a, b, and c are constants.

(a) (8%) If r(x)=0 and $b^2=4ac$, find the solution y(x).

(b) (8%) If r(x)=0 and $b^2=3ac$ (where ac>0), find the solution y(x).

(c) (9%) If $r(x)=de^{fx}$ (where d and f are constants), and $b^2=4ac$, $f=-\frac{b}{2a}$, find the solution y(x).

4. (25%)

Given the line integral $\int_{(1,1.5)}^{(2,0.5)} (kx^2\sin\pi y dx+\pi x^3\cos\pi y dy)$, where k is a constant.

(a) (10%) Determine the value of k that the line integral is independent of path.

(b) (15%) Evaluate the path independent integral.