

# 淡江大學八十九學年度碩士班招生考試試題

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系別：土木工程學系

科目：工程數學

本試題共 / 頁

1. Solve the first-order differential equation  $(x^3 + 1)y' + 3x^2y = x^2 - 1$  (15%)

2. Let the differential operator  $\alpha$  represent  $\frac{d^2}{dx^2} + \frac{1}{x} \frac{d}{dx}$ , answer the following;

(a) Please expand the operator  $\alpha^2 \phi = \alpha \alpha \phi = 0$  to a differential equation. (10%)

(b) Please solve the equation you obtain in (a). (25%)

3.  $A = \begin{bmatrix} k & 0 & -1 \\ 1 & -1 & 0 \\ 0 & 1 & k \end{bmatrix}$ , answer the following questions;

(a) Please explain the meanings of "A is Positive-Definite" and "A is Negative-Definite", respectively. (10%)

(b) If  $k = -1$ , please show whether A is positive-definite or negative-definite or none. (15%)

4. Use Laplace Transformation (with respect to  $t$ ) to solve the partial differential equation (25%)

$y_{tt}(x, t) = a^2 y_{xx}(x, t) - g$ , where  $a$  and  $g$  are constants;

and  $y(x, t)$  satisfies the boundary conditions  $y(x, 0) = y_t(x, 0) = 0$ ,

$$y(0, t) = 0, \lim_{x \rightarrow \infty} y_x(x, t) = 0$$