

淡江大學八十七學年度碩士班入學考試試題

系別：土木工程學系

科目：工程數學

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請按照題目順序作答

(I) Find differential equations for which the following equations are general solutions. (10%)

(i) $y = (1 + 3x)e^{2x}$ (10%)

(ii) $y = c_1\sqrt{x} + c_2\frac{1}{\sqrt{x}}$

(II) Find general solution of the following differential equation. (10%)

$$2(3z+1)^2 y'' + 21(3z+1)y' + 18y = 0$$

(III) Let $[C] = \begin{bmatrix} A+ia & D+id \\ B+ib & E+ie \end{bmatrix}$ Give the following definitions in terms of A,B,D,E

and a,b,d,e. (10%)

(i) $[C]$ is a symmetric matrix

(ii) $[C]$ is a skew-symmetric matrix

(iii) $[C]$ is a Hermite matrix

(iv) $[C]$ is a skew-Hermite matrix

(IV) Let $[A] = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, show that $[A]$ can be expressed as the sum of a symmetric

matrix and a skew-symmetric matrix. (10%)

(V) Use matrix method to find general solution of the following system of differential equations. (20%)

$$\frac{dy_1}{dt} = -4y_1 + y_2 + y_3$$

$$\frac{dy_2}{dt} = 2y_2 - 5y_3$$

$$\frac{dy_3}{dt} = -4y_3$$

(VI) Show that $\int_C (y + yz)dx + (x + 3z^3 + xz)dy + (9yz^2 + xy - 1)dz$ is independent

of any path from (1,1,1) to (2,1,4), then, evaluate the integration. (20%)

(VII) Apply Laplace transform to solve the following partial differential equation. (20%)

$$\frac{\partial w(x,t)}{\partial x} + x \frac{\partial w(x,t)}{\partial t} = 0; \quad t \geq 0$$

$$w(x,0) = 0, \quad w(0,t) = t$$