1. Find the integrating factor of the equation : $x \sin y \; dy = (2 \cos y + 4x^2) \; dx$ and solve it. (20%)

2. Solve the initial value problems: $y'' + 2y' - 8y = e^{-4x} + x$, $\quad y(0) = 1, y'(0) = 0$ (20%)

3. Use the "Laplace Transform" to solve the initial value problems: $y'' + 6y' + 9y = t + 2e^{-3t}$, $y(0) = 1, y'(0) = 0$ (20%)

4. Given $\mathbf{A} = \begin{bmatrix} -2 & 3 & 1 \\ 1 & 11 & 3 \\ -7 & -27 & -7 \\ 1 & 36 & 10 \end{bmatrix}$; $\mathbf{b} = \begin{bmatrix} 1 \\ 5 \\ -13 \\ 16 \end{bmatrix}$

(a) Find the rank of $\mathbf{A}$ using Gauss Elimination Method. (10%)

(b) Find the dimension of row space of $\mathbf{A}$. (5%)

(c) Does the equation $\mathbf{Ax=b}$ have solutions? (10%) If yes, write down the solution. (5%)

(d) Write down the vector in general form in the null space. (10%)