

系別：土木工程學系

科目：工程數學

准帶項目請打「V」	
✓	簡單型計算機

本試題共 / 頁，4 大題

1. Solve the following initial value problems. (25%)

(1)  $x^2 y'' + xy' - y = 26$  ;  $y(0.8) = 0, y'(0.8) = -7.5$

(2)  $y'' - 2y + y = e^x$  ;  $y(0) = 0, y'(1) = 0$

2. Use method of Laplace transform to solve the given set of differential equations. (25%)

$\frac{dx}{dt} - 6x + 3y = 8e^t$  ;  $x(0) = -1, y(0) = 0$

$\frac{dy}{dt} - 2x - y = 4e^t$

3. In matrix structural analysis, system stiffness matrix is obtained by  $[K] = [a]^T [k] [a]$ .

If system stiffness matrix can be expressed in the form of  $[K] = \begin{bmatrix} [K_{ff}] & [K_{sf}] \\ [K_{sf}] & [K_{ss}] \end{bmatrix}$ ,

and  $[a] = \begin{bmatrix} -1 & 0 & 0 & 1 & 0 & 1 & 0 \\ -1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 & 1 & 0 & 1 \end{bmatrix}$ ,  $[k] = \begin{bmatrix} 4 & 2 & 0 & 0 & 0 & 0 \\ 2 & 4 & 0 & 0 & 0 & 0 \\ 0 & 0 & 4 & 2 & 0 & 0 \\ 0 & 0 & 2 & 4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 4 & 2 \\ 0 & 0 & 0 & 0 & 2 & 4 \end{bmatrix}$ . In which,  $[K_{ff}]$  is the

system stiffness matrix of free coordinates and has dimension of  $(3 \times 3)$ , i.e.,

$[K_{ff}] = \begin{bmatrix} K_{11} & K_{12} & K_{13} \\ K_{21} & K_{22} & K_{23} \\ K_{31} & K_{32} & K_{33} \end{bmatrix}$ . Use the smallest sub-matrix of  $[a]$  and  $[k]$  to calculate

$[K_{ff}]$ , then find the inversed matrix  $[K_{ff}]^{-1}$ . (25%)

4. For the given matrix, determine the eigen values and eigen vectors. (25%)

$\begin{bmatrix} 1 & -3 & 9 \\ 0 & -5 & 18 \\ 0 & -3 & 10 \end{bmatrix}$