淡江大學八十八學年度碩士班招生考試試題

系別：土木工程學系　　科目：工程數學　　本試題共 / 頁

1. (25%)

(a) (13%) Solve the differential equation

$$x^2y'' - 2xy' + 2y = \frac{24}{x^2}$$

(b) (12%) Sketch the following functions and find their Laplace transforms:

(i) (6%) $(t-1)^2 u(t-1)$　　(ii) (6%) $u(t-\frac{\pi}{2})\sin t$

where $u(t)$ is the unit step function (also known as the Heaviside function).

2. (25%) Use the laplace transforms to solve an intial value problem

$$y'' - y = -2\sin t + \delta(t-1) \quad ; \quad y(0) = 0,\ y'(0) = 2$$

where $\delta(t-1)$ is the Dirac delta function.

3. (25%) Given a matrix

$$A = \begin{bmatrix} -1 & 0 & 5 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{bmatrix}$$

(1) (12%) Find the eigenvalues and eigenvectors of the matrix.

(2) (13%) Apply the diagonalization formula: $D = X^{-1}AX$ to diagonalize the matrix A. You are required to show the detailed procedure of expanding $(X^{-1}AX)$ to obtain the diagonal matrix D.

4. (25%) Given a line integral $\int_{(0,0,0)}^{(1,1,1)} e^{xyz}(yz\,dx + xz\,dy + xy\,dz)$.

(1) (15%) Examine to show if the above integral is dependent or independent of the path of integration.

(2) (10%) Determine the value of integration.