

系別：電機工程學系
資訊工程學系

二年級

科目：微積分

本試題共 / 頁

務必寫出演算過程，否則不予計分：

一、每一小題佔 7%：

(a) $\lim_{x \rightarrow 1} \frac{x^3 - x^2 + x - 1}{x^3 - 1} = ?$

(b) If $f(x) = \int_x^{x^2} \sqrt{1+t^2} dt$, then $f'(1) = ?$

(c) Given $f'(x) = \frac{1}{x} + \cos \frac{\pi}{2}x$ and $f(1) = \frac{2}{\pi}$, find $f(x)$.

(d) Find the area of the region enclosed by $y = |x|$ and $y = x^2$.

(e) $\lim_{x \rightarrow 0^+} x^x = ?$

(f) $\lim_{x \rightarrow 0} \frac{|x|}{x} = ?$

(g) Evaluate the double integral $\iint_D xy dx dy$,

where $D = \{ (x, y) : 0 \leq x \leq 1, x^2 \leq y \leq x \}$.

(h) $\int \frac{dx}{x^2 - 1} = ?$

(i) $\int_{\frac{1}{2}}^1 x \ln x dx = ?$

(j) If $f(x, y) = \sin(xy) + e^{xy}$, then $f_{xy}(1, \pi) = ?$

二、If f and g are continuous functions on $[a, b]$ and both f' and g' exist on (a, b) , show that there exists $c \in (a, b)$ such that

$$f'(c)[g(b) - g(a)] = g'(c)[f(b) - f(a)], \quad (10\%)$$

三、Define $d(x, y)$ on $\mathbb{R} \times \mathbb{R}$ by

$$d(x, y) = \frac{|x - y|}{1 + |x - y|},$$

where $x, y \in \mathbb{R}$ (the set of real numbers). Show that $d(x, y) \leq d(x, z) + d(z, y)$

for all $x, y, z \in \mathbb{R}$. (10%)

四、Find the sum of $\sum_{n=0}^{\infty} \frac{n^2}{n!}$. (10%)