

須書寫演算過程，祇寫答案不給分。

1. Find the following:

(a)  $\lim_{x \rightarrow 1} \frac{x^3 - 1}{x^4 - 1}$

(b)  $\lim_{x \rightarrow 0} \left( \frac{1}{x} - \frac{1}{\ln(1+x)} \right)$

(c)  $\lim_{n \rightarrow \infty} a_n$ , where  $a_1 = \sqrt{2}$  and  $a_{n+1} = \sqrt{2 + a_n}$  for  $n \geq 1$ .

(25%)

(d)  $f'(0)$ , where  $f(x) = [1 + (1+x)^2]^3$

(e)  $g'(1)$ , where  $g(x) = \int_0^{x^2} \sqrt{1+t^3} dt$ .

2. Find the following:

(a)  $\lim_{n \rightarrow \infty} \frac{\sqrt{1} + \sqrt{2} + \dots + \sqrt{n}}{n\sqrt{n}}$

(b)  $\int_0^1 \frac{x}{1+x^4} dx$

(c)  $\int_0^1 \ln x dx$

(d)  $f(x)$ , given  $f'(x) = x + \frac{1}{x}$  and  $f(1) = \frac{3}{2}$ .

(25%)

(e) the sum of the series  $\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$ .

3. Let

$$f(x) = \begin{cases} x^2 \sin \frac{1}{x} & \text{if } x \neq 0, \\ 0 & \text{if } x = 0. \end{cases}$$

(10%)

Find  $f'(0)$ . Is  $f$  continuous at  $x=0$ ? Justify your answer.

4. Find the total length of the curve  $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$ ,  $a > 0$ .

(10%)

5. (a) Find the power series for  $\frac{\sin x}{x}$ .

(b) Express  $\int_0^1 \frac{\sin x}{x} dx$  as a sum of an infinite series.

(10%)

6. Let

$$f(x,y) = \begin{cases} \frac{xy}{x^2 + y^2} & \text{if } (x,y) \neq (0,0), \\ 0 & \text{if } (x,y) = (0,0). \end{cases}$$

(10%)

(a) Find  $f_x(0,0)$  and  $f_y(0,0)$ .

(b) Is  $f$  continuous at  $(0,0)$ ? Justify your answer.

7. Find the double integral

$$\iint_R xy^2 dA,$$

(10%)

where  $R = \{ (x,y) : 0 \leq x \leq 1, 0 \leq y \leq 1 \}$ .