淡江大學95學年度碩士班招生考試試題

系別：物理學系　　　　科目：近 代 物 理

准帶項目請打「V」	
	簡單型計算機

本試題共 / 頁

1. (25 points) Answer the following questions. Be concise and to the point.
   (a) (5 points) Discuss the photoelectric effect and its significance.
   (b) (5 points) Explain quantum tunneling. Give two physical examples of this effect.
   (c) (5 points) Discuss the Stern-Gerlach experiment and its significance.
   (d) (5 points) Describe the difference between bosons and fermions? Give two examples of each particles.
   (e) (5 points) Explain in pair-annihilation processes why we observe $e^- + e^+ \to 2\gamma$ but not $e^- + e^+ \to \gamma$.

2. (15 points) Consider a non-relativistic electron moving in the plane perpendicular to a constant, uniform magnetic field. Use Bohr's quantization rule to calculate the energy levels of the electron.

3. (20 points) Let the potential of a one-dimensional potential well be given by

$$V(x) = \begin{cases} 0 & 0 < x < L, \\ \infty & \text{otherwise.} \end{cases}$$

   (a) (10 points) Find the eigenvalues and eigenfunctions of the time-independent Schrödinger equation.
   (b) (5 points) Consider two noninteracting electrons in the potential well. What are the energy and the total wave function of the lowest energy state if the two electrons are in the *same* spin state?
   (c) (5 points) Continue from (b), what are the energy and the total wave function of the lowest energy state if the two electrons are in *different* spin states?

4. (20 points) Consider a system which is described by the state

$$|\Psi\rangle = N\left[\sqrt{3}\,|1,1\rangle - |1,0\rangle + 2\,|1,-1\rangle\right],$$

   where $|\ell, m\rangle$ are normalized eigenstates of the angular momentum operators L^2 $(= L_x^2 + L_y^2 + L_z^2)$ and L_z with the corresponding eigenvalues $\ell(\ell+1)\hbar^2$ and $m\hbar$, respectively, and $N > 0$ is a normalization constant.
   (a) (5 points) Find the value of N so that $|\Psi\rangle$ is normalized.
   (b) (5 points) Find $L_+|\Psi\rangle$ and $L_-|\Psi\rangle$, where $L_\pm = L_x \pm iL_y$.
   (c) (5 points) Calculate the expectation values of L_x and L^2 in the state $|\Psi\rangle$.
   (d) (5 points) If L_z is measured, what values will be obtained? With what probabilities?

5. (20 points) Let us try to understand the physics of neutrino oscillations. Consider a quantum mechanical two-state system whose basis states we shall call $|\nu_e\rangle$ and $|\nu_\mu\rangle$ (corresponding to the electron-type and muon-type neutrinos produced in weak decay processes). At time $t = 0$ the system is in the state $|\nu_e\rangle$. Suppose the Hamiltonian of the system in the $\{|\nu_e\rangle, |\nu_\mu\rangle\}$ basis can be written as

$$H = \begin{pmatrix} M & -i\Delta \\ i\Delta & M \end{pmatrix},$$

   where $M \gg \Delta > 0$ are constants.
   (a) (5 points) Find the eigenvalues E_j and the normalized eigenstates $|\nu_j\rangle$ of H. Here $j = 1, 2$.
   (b) (5 points) Find the state of the system at time $t > 0$.
   (c) (5 points) Calculate the probability that the system is in the state $|\nu_\mu\rangle$ as a function of time.
   (c) (5 points) The above analysis provides a simple explanation of the oscillation $\nu_e \to \nu_\mu$. Given that neutrino oscillations have been observed experimentally, what properties of neutrinos can be drawn?