淡江大學八十八學年度碩士班招生考試試題

系別：物理學系　　科目：近代物理　　本試題共 一 頁

1. Use the Bohr quantization rules to calculate the energy levels for a harmonic oscillator, for which the energy function is

$\frac{p^2}{2m} + \frac{1}{2} m\omega^2 r^2$, where r is the radial distance, p the momentum, m and ω are the mass and angular frequency of the orbiting particle. Restrict yourself to circular orbits. (20%)

2. According to Yukawa, the nuclear force arises through the emission of a new quantum (called pion) by one of the nucleons, and its absorption by the other. If the range of the nuclear force is R, give an estimate of the mass of the pion. (20%)

3. A particle in free space is initially in a wave packet described by

$$\psi(x) = \left(\frac{\alpha}{\pi}\right)^{\frac{1}{4}} e^{-\frac{\alpha}{2}x^2} \qquad (\alpha > 0).$$

What is the probability that its momentum is in the range $(p, p+dp)$? (Useful formula: $\int_{-\infty}^{\infty} dx\, e^{-\alpha x} = \sqrt{\frac{\pi}{\alpha}}$). (20%)

4. The average value of momentum p of a particle in a state $\psi(x)$ is given by

$$\langle p \rangle = \int_{-\infty}^{\infty} dx\, \psi^*(x) \frac{\hbar}{i} \frac{\partial}{\partial x} \psi.$$

i) show that $\langle p \rangle$ is real;
ii) if $\psi(x)$ is a real function, show that $\langle p \rangle = 0$. (20%)

5. An operator A, corresponding to an observable α, has two normalised eigenfunctions ϕ_1 and ϕ_2, with eigenvalues a_1 and a_2. An operator B, corresponding to an observable β, has normalised eigenfunctions χ_1 and χ_2, with eigenvalues b_1 and b_2. The eigenfunctions are related by

$$\phi_1 = \frac{1}{\sqrt{13}}(2\chi_1 + 3\chi_2), \quad \phi_2 = \frac{1}{\sqrt{13}}(3\chi_1 - 2\chi_2).$$

α is measured and the value a_1 is obtained. If β is then measured and then α again, show that the probability of obtaining a_1 a second time is $97/169$. (20%)