淡江大學八十七學年度碩士班入學考試試題

系別： 物理系　　　科目： 近代物理

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1. Explain briefly the following terms: (a) Time dilation, (b) Photoelectric effect, (c) Fermi-Dirac distribution, (d) LS coupling, (e) Fine structure.　(20 %)

2. In thermal equilibrium, the blackbody radiation spectral energy density is given by

$$u(f,T)=\frac{8\pi hf^3}{c^3}\frac{1}{e^{hf/kT}-1},$$

where f is the oscillation frequency of a radiation mode, and T is the temperature. Identify the average energy per mode from the above formula, and show how to derive it.　(20 %)

3. A one-dimensional simple-harmonic quantum oscillator of mass m is bound by the potential: $V(x)=m\omega^2x^2/2$, where ω is the angular frequency (of the classical oscillation). Assume that the oscillator stays in an energy eigenstate described by the wave function: $\psi(x)= C\,exp(-\alpha x^2)$. (a) Find the constant α and the eigenenergy, E, in terms of m and ω by using the Schroedinger equation: $\psi''(x)+[2m(E-m\omega^2x^2/2)/\hbar^2]\,\psi(x)=0$;
(b) normalize the wave function; and then, (c) calculate the uncertainties in the oscillator's position and momentum, and also their product ($\Delta x\,\Delta p$).　(20 %)

4. A quantum particle of mass m is bound by the one-dimensional square-well potential:
$V(x)=-V_0$, if $|x|<L$, and $V(x)=0$, if $|x|>L$, where V_0 is a positive constant.
(a) Derive the equations for the energy of the bound energy eigenstates?
(b) If $(n-1)\,\pi\hbar<\sqrt{8mV_0}\,L=\leq n\,\pi\hbar$, where n is a positive integer; then, how many bound eigenstates can exist in the system?
(c) Is $E=0$ an acceptable eigenenergy? Justify your answer.　(20 %)

5. The quantum state of a system of hydrogen atoms is described by the wave function $\Psi=\Psi_{100}+2\Psi_{210}+2\Psi_{21\text{-}1}$, where Ψ_{nlm}'s are the well-known hydrogenic eigenwave functions. Normalize the wave function, and then, calculate the expectation values of the state's energy, $\bar{L}^2$ ($\bar{L}$= orbital angular momentum), and L_z.　(20%)