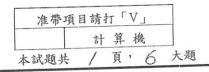
## 淡江大學 99 學年度碩士班招生考試試題

系別:物理學系

科目:物理數學



- 1. Prove the identity  $(\vec{C} \times \vec{D}) \cdot (\vec{E} \times \vec{F}) = (\vec{C} \cdot \vec{E})(\vec{D} \cdot \vec{F}) (\vec{C} \cdot \vec{F})(\vec{D} \cdot \vec{E})$  for the vectors in the 3-dimensional Cartesian coordinate system. (20%)
- 2. For A a non-Hermitian operator, show that
  (a)  $A + A^+$  and  $i(A A^+)$  are Hermitian operators. (10%)
  - (b) show that, by applying the result above, every non-Hermitian operator may be written as a linear combination of two Hermitian operators. (10%)
- 3. (a) Show that, by multiplying  $I(x) = e^{K(x)}$  with  $K(x) = \int_{-x}^{x} P(t)dt$ , called integration factor, to the first-order, linear ODE (dy/dx) + P(x)y = Q(x), we can express the solution as  $y(x) = e^{-K(x)} [\int_{-x}^{x} e^{K(s)} Q(s) ds + \text{constant}]$ . (10%)
  - (b) With the condition  $y(\pi) = 1$ , find the solution of the ODE  $x(dy/dx) 2y = x^3 \cos 4x$ . (10%)
- 4. Apply the residue theorem to evaluate  $\int_{-\infty}^{\infty} \frac{\cos(2x)}{1+x^3} dx$ . (10%)
- 5. Find the Fourier transform of the triangular pulse

$$f(x) = \begin{cases} k(1 - a|x|), |x| < \frac{1}{a} \\ 0, |x| > \frac{1}{a} \end{cases}$$
 (10%)

6. Apply the Fourier transformation  $p(k,\tau) = (2\pi)^{-1/2} \int_{-\infty}^{\infty} q(x,\tau) e^{ikx} dx$  to show that the solution of the one-dimensional Fermi age equation  $[\partial q(x,\tau)/\partial \tau] = [\partial^2 q(x,\tau)/\partial x^2]$  (here, q is the number of neutrons that slow down, falling below some given energy per second per unit volume and  $\tau$  in a measure of the energy loss) is  $q(x,\tau) = \frac{S}{\sqrt{4\pi\tau}} e^{-x^2/4\tau}$  with  $S\delta(x) = q(x,0)$  a plane source of neutrons at x=0 emitting S neutrons per unit area per second. (20%)