淡江大學99學年度碩士班招生考試試題

系別：物理學系　　　　　科目：物　理　數　學

准帶項目請打「V」	
	計算機

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1. Prove the identity $(\vec{C}\times\vec{D})\cdot(\vec{E}\times\vec{F})=(\vec{C}\cdot\vec{E})(\vec{D}\cdot\vec{F})-(\vec{C}\cdot\vec{F})(\vec{D}\cdot\vec{E})$ for the vectors in the 3-dimensional Cartesian coordinate system. (20%)

2. For A a non-Hermitian operator, show that
   (a) $A+A^{+}$ and $i(A-A^{+})$ are Hermitian operators. (10%)

   (b) show that, by applying the result above, every non-Hermitian operator may be written as a linear combination of two Hermitian operators. (10%)

3. (a) Show that, by multiplying $I(x)=e^{K(x)}$ with $K(x)=\int^{x}P(t)dt$, called *integration factor*, to the first-order, linear ODE $(dy/dx)+P(x)y=Q(x)$, we can express the solution as $y(x)=e^{-K(x)}[\int^{x}e^{K(s)}Q(s)ds+\text{constant}]$. (10%)

   (b) With the condition $y(\pi)=1$, find the solution of the ODE $x(dy/dx)-2y=x^{3}\cos 4x$. (10%)

4. Apply the residue theorem to evaluate $\int_{-\infty}^{\infty}\frac{\cos(2x)}{1+x^{3}}dx$. (10%)

5. Find the Fourier transform of the triangular pulse

$$f(x)=\begin{cases}k(1-a|x|), & |x|<\frac{1}{a}\\ 0, & |x|>\frac{1}{a}\end{cases}. \quad (10\%)$$

6. Apply the Fourier transformation $p(k,\tau)=(2\pi)^{-1/2}\int_{-\infty}^{\infty}q(x,\tau)e^{ikx}dx$ to show that the solution of the one-dimensional Fermi age equation $[\partial q(x,\tau)/\partial\tau]=[\partial^{2}q(x,\tau)/\partial x^{2}]$ (here, q is the number of neutrons that slow down, falling below some given energy per second per unit volume and τ is a measure of the energy loss) is $q(x,\tau)=\frac{S}{\sqrt{4\pi\tau}}e^{-x^{2}/4\tau}$ with $S\delta(x)=q(x,0)$ a plane source of neutrons at $x=0$ emitting S neutrons per unit area per second. (20%)