

系別：物理學系

科目：物理數學

准帶項目請打「V」

簡單型計算機

本試題共 1 頁，5 大題

1. (1) Verify the expansion of the vector product  $\vec{A} \times (\vec{B} \times \vec{C}) = \vec{B}(\vec{A} \cdot \vec{C}) - \vec{C}(\vec{A} \cdot \vec{B})$  by direct expansion in Cartesian coordinates. (10%)
- (2) With vector  $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$  ( $\hat{i}, \hat{j}, \hat{k}$  the unit vectors in Cartesian coordinates) and its magnitude  $r = \sqrt{x^2 + y^2 + z^2}$ , show that

$$\vec{\nabla} \cdot [\vec{r}f(r)] = 3f(r) + r \frac{df(r)}{dr}. \quad (10\%)$$

2. If A and B are Hermitian matrices,
- (1) show that  $(AB+BA)$  and  $i(AB-BA)$  are also Hermitian. (10%)
- (2) in the case A and B have the same eigenvalues, show that A and B are related by a unitary similarity transformation. (10%)
3. (1) Show that, with  $z = x + iy$  a complex number,  $\tan^{-1} z = (i/2) \ln[(i+z)/(i-z)]$  (10%);
- (2) Applying the residue theorem, show that  $\int_{-\infty}^{\infty} \frac{x^2}{1 - 2x^2 \cos(2\theta) + x^4} dx = \frac{\pi}{2 \sin \theta}$  (15%)

4. Show that a triangular wave represented by  $f(x) = \begin{cases} x, & 0 < x < \pi \\ -x, & -\pi < x < 0 \end{cases}$  can be expanded by a Fourier series,  $f(x) = \frac{\pi}{2} - \frac{4}{\pi} \sum_{n=1,3,5,\dots} \frac{\cos nx}{n^2}$ . (15%)

5. (1) Show that the Laplace transform  $L\{\cos at\} = s/(s^2 + a^2)$  with  $s > 0$ . (5%)
- (2) Applying the Laplace transform, solve the equation for the simple harmonic oscillator with a mass  $m$  under the influence of an ideal spring with spring constant  $k$ ,

$$m \frac{d^2 x(t)}{dt^2} = -kx(t)$$

with the initial conditions  $x(0) = x_0$  and  $[dx(0)/dt] = 0$ . (15%)