淡江大學96學年度碩士班招生考試試題

系別：物理學系　　　　　　科目：物 理 數 學

1. Show the following two identities

    (1) $(\vec{A}\times\vec{B})\cdot(\vec{C}\times\vec{D}) = (\vec{A}\cdot\vec{C})(\vec{B}\cdot\vec{D}) - (\vec{A}\cdot\vec{D})(\vec{B}\cdot\vec{C})$　　(10%)

    (2) $(\vec{r}\times\vec{\nabla})\cdot(\vec{r}\times\vec{\nabla})f = [r^2\nabla^2 - r^2\dfrac{\partial^2}{\partial r^2} - 2r\dfrac{\partial}{\partial r}]f$,

    where vector $\vec{r} = x\,\hat{x} + y\,\hat{y} + z\,\hat{z}$, its magnitude $r = \sqrt{x^2 + y^2 + z^2}$

    and f a scalar function. (10%)

2. (a) Two matrices A and B are each Hermitian. What is the necessary and sufficient condition for their product AB to be also Hermitian? (10%)

    (b) Show that a Hermitian Matrix remains Hermitian under unitary similarity transformations. (10%)

3. Find the Fourier transform of the Gaussian probability function $f(x) = Ne^{-\alpha x^2}$ with N, α constant. (15%) Discuss how the shapes of $f(x)$ and its Fourier transform change as the value of α is increased. (5%)

4. Evaluate

    (1) $\displaystyle\int_{-\infty}^{\infty} g(x)\delta(x^2 - x - 6)dx = ?$, where $\delta(x)$ the Dirac delta function. (10%)

    (2) $\displaystyle\int_{-\infty}^{\infty} \dfrac{dx}{4x^2 + 2x + 1} = ?$　(10%)

5. The motion of a body (with mass m) falling in a resisting medium may be described by the differential equation

    $$m\dfrac{dv}{dt} = mg - bv ,$$

    where $v(t)$ the velocity of the body, g the constant acceleration due to gravity, and b the retarding-force constant. With the initial condition $v(0)=0$, find $v(t)$. (20%)