

淡江大學 95 學年度碩士班招生考試試題

27

系別：物理學系

科目：物 理 數 學

准帶項目請打「V」	
	簡單型計算機

本試題共 / 頁

1. Given $f(x) = 1, -\frac{\pi}{2} < x < \frac{\pi}{2}; 0, \frac{\pi}{2} < x < \frac{3\pi}{2}$, where $f(x+2\pi) = f(x)$. Expand $f(x)$ in Fourier series and evaluate $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots = ?$. (20 points)

2. (a) If A and B are Hermitian matrices, show that $(AB + BA)$ and $i(AB - BA)$ are also Hermitian matrices. (10 points)

(b) Two Hermitian matrices A and B have the same eigenvalues. Show that A and B are related by a unitary similarity transformation. (10 points)

3. (a) Given $\vec{A} = x^2\hat{i} + y^2\hat{j} + z^2\hat{k}$, calculate directly $\int \vec{A} \cdot d\vec{\sigma}$ over the whole surface of the cube with four of its vertices at $(0, 0, 0), (0, 0, 1), (0, 1, 0)$ and $(1, 0, 0)$. (10 points)

(b) Evaluate the same integral by using of the divergence theorem. (10 points)

4. Solve the following 1st-order ordinary differential equation. (10 points)

$$xy' + y = xy^3.$$

5. Evaluate $I = \int_{-\infty}^{\infty} \frac{x \sin x}{x^2 + 1} dx = ?$ (15 points)

6. (a) Show that $f(x) = \frac{1}{\pi} \frac{\epsilon}{x^2 + \epsilon^2}$ approaches to $\delta(x)$ as $\epsilon \rightarrow 0^+$. (8 points)

(b) By using of $\Gamma(z) = \int_0^{\infty} e^{-t} t^{z-1} dt (z > 0)$, evaluate $\int_0^{\infty} \sqrt{x} e^{-x^2} dx = ?$ (7 points)