淡江大學九十四學年度碩士班招生考試試題

系別：物理學系　　　　科目：物　理　數　學

准帶項目請打「∨」
簡單型計算機

本試題共 / 頁

1. Find the solution of the differential equation

$$a(d^2x/dt^2) + b(dx/dt) + cx = g\cos\alpha t ,$$

(i) if $b = 0$, $g = 0$, and $ac \neq 0$.　(10%)
(ii) if $g = 0$, and $abc \neq 0$.　(10%)
(iii) $abcg\alpha \neq 0$.　(10%)

2. Show that

$$\nabla(A \cdot B) = A \times (\nabla \times B) + B \times (\nabla \times A) + (A \cdot \nabla)B + (B \cdot \nabla)A.$$

(10%)

3. Find the value of the integral $\oint_S A \cdot da$, where $A = xi - yj + zk$ and S is the closed surface defined by the cylinder $c^2 = x^2 + y^2$. The top and bottom of the cylinder are at $z = d$ and 0, respectively. (10%)

4. Find the eigenvalues and the corresponding eigenvectors of the matrix
$$\begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix} .$$
　(20%)

5. Obtain the Fourier series representing the function
$$F(t) = \begin{cases} 0 & -2\pi/\omega < t < 0 \\ \sin\omega t & 0 < t < 2\pi/\omega . \end{cases}$$
(10%)

6. Show that $\int_0^\infty (\sin x/x)dx = \pi/2$.　(10%)

7. Legendre's differential equation may be written in the form

$$(1-x^2)[d^2P_n(x)/dx^2] - 2x[dP_n(x)/dx] + n(n + 1)P_n(x) = 0.$$

Show that $\int_{-1}^{1} P_m(x) P_n(x)dx = 0$, for $m \neq n$.　(10%)