淡江大學九十二學年度碩士班招生考試試題

系別：物理學系　　　科目：物　理　數　學

准帶項目請打「○」否則打「×」	
簡單型計算機	×

1. Find the eigenvalues and corresponding eigenvectors of the following matrix (15 points)

$$\begin{pmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{pmatrix}.$$

2. (a) Show that $f(x)=\frac{1}{\pi}\frac{\varepsilon}{x^2+\varepsilon^2}$ approaches to 1-D $\delta(x)$ as $\varepsilon \to 0^+$. (8 points)

   (b) By using of $\Gamma(z)=\int_0^\infty e^{-t}t^{z-1}dt\,(z>0)$, evaluate $\int_0^\infty \sqrt{x}e^{-x^2}dx=?$ (7 points)

3. Use the method of contour integration to calculate $I=\int_0^\infty \frac{\cos bx}{x^2+a^2}dx$, where $b>0$.

   (15 points)

4. Solve the following differential equation (15 points)

$$y''+2y'+5y=16e^x+\sin 2x.$$

5. Given $f(x)=x(\pi-x), 0<x<\pi,$

   $x(\pi+x), -\pi<x<0,$

   expand $f(x)$ in Fourier series and show that $\sum_{n=odd}(-1)^{(n-1)/2}n^{-3}=\frac{\pi^3}{32}$. (20 points)

6. (a) Write down mathematical expression of Gauss's theorem, Stokes's theorem, and Green's theorem, respectively. (Note: you have to specify the meaning of each notation shown in the expression.) (8 points)

   (b) If $\vec{F}=x\hat{i}+y\hat{j}$, calculate $\iint \vec{F}\cdot\hat{n}d\sigma$ over the part of the surface $z=4-x^2-y^2$ that is above the x-y plane. (12 points)