淡江大學八十七學年度碩士班入學考試試題

系別： 物理系　　科目： 物理數學

本試題共 2 頁

1. Find the general solution for the following differential equations:

(1) $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 4y = 3 + 2e^x$

(2) $x^2\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = x^3 \ln x$

(20%)

2. Consider the surface $z^2 = x^2 - y^2$. Find the normal vector and the tangential plane to this surface at point (1,1,0). (20%)

3. The definition of the convolution of $f(x)$ with $g(x)$ is denoted $f(x)*g(x)$, and is given by $f(x)*g(x) = \int_0^x f(x')g(x-x')dx'$, and the Laplace transform of the convolution is

$$\mathcal{L}\{f(x)*g(x)\} = \mathcal{L}\{f(x)\}\mathcal{L}\{g(x)\} = F(s)G(s)$$

where $\mathcal{L}\{f(x)\} = F(s)$, $\mathcal{L}\{g(x)\} = G(s)$ (20%)

Find a function $f(x)$ satisfying $f(x) = 2x^2 + \int_0^x f(x-x')e^{-x'}dx'$

【 The Laplace transform of x^n and e^{ax} are

$\mathcal{L}\{x^n\} = \frac{n!}{s^{n+1}}, n = 1,2,3.....$, $\mathcal{L}\{e^{ax}\} = \frac{1}{s-a}$ 】

4. The moments of inertia of a uniform square plate of length a about the x, y and z axes chosen as shown in Fig.1, is given

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by $I=\begin{pmatrix} \frac{1}{3}ma^2 & -\frac{1}{4}ma^2 & 0 \\ -\frac{1}{4}ma^2 & \frac{1}{3}ma^2 & 0 \\ 0 & 0 & \frac{2}{3}ma^2 \end{pmatrix}$

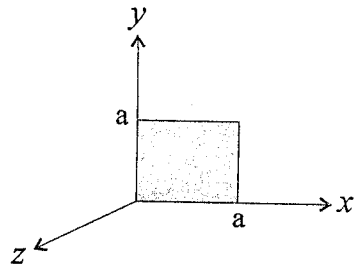

Fig.1

Find the (a) principal moments of inertia (eigenvalues) and (b) the directions of the principal axes for the plate (eigenvectors). (20%)

5. Solve the partial differential equation with some boundary conditions.

$$\frac{\partial^2 f}{\partial x^2}+\frac{\partial^2 f}{\partial y^2}=0$$

(20%)

$$\frac{\partial f}{\partial x}\Big|_{(0,y)}=0 \quad , \quad \frac{\partial f}{\partial x}\Big|_{(L,y)}=0 \quad , \quad u(x,0)=0$$