

系別：電機工程學系三年級

科目：工 程 數 學

可否使用計算機			
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本試題共 / 頁

- Solve the following differential equation:  $x^2 y'' - 5xy' + 8y = 2 \ln(x)$ , where  $y'' = d^2 y(x)/dx^2$  and  $y' = dy(x)/dx$ . (15 points)
- Prove  $\|F + G\|^2 + \|F - G\|^2 = 2(\|F\|^2 + \|G\|^2)$ , where  $F$  and  $G \in \mathbb{R}^n$ . (15 points)
- Consider the following symmetric matrix:  $A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ . Find a matrix  $P$  that diagonalizes  $A$ . (20 points)
- Plot the following time functions: (a)  $y_1 = t^2$ , (b)  $y_2 = t^2 H(t-3)$ , (c)  $y_3 = (t-3)^2 H(t-3)$ , where  $H(t-3) = \begin{cases} 0, & \text{if } t < 3 \\ 1, & \text{if } t \geq 3. \end{cases}$  Then find the corresponding Laplace Transforms, i.e.,  $Y_1(s), Y_2(s)$  and  $Y_3(s)$ . (20 points)
- Prove that  $\nabla \cdot (\nabla \times F) = 0$ , where  $F$  is a continuous vector field whose components have continuous first and second partial derivatives. (15 points)
- For a function  $f(z) = z^5 + 1$ , find the values of  $u$  and  $v$  such that  $u + iv = f(3 + 2i)$ . (15 points)