1. (10%) Show that $\neg(p \to q)$ and $(p \text{ AND } \neg q)$ are logically equivalent.

2. (5%) Prove or disprove that $\lceil x + y \rceil = \lceil x \rceil + \lceil y \rceil$

3. (10%) Let x be a real number with $|x| < 1$. Find $\sum_{n=0}^{\infty} x^n$.

4. (20%) Find $f(2), f(3), f(4)$, and $f(5)$ if f is defined recursively by $f(0) = f(1) = 1$ and for $n = 1$, 2, 3, 4, ...
   a. $f(n+1) = f(n) - f(n-1)$
   b. $f(n+1) = f(n) \times f(n-1)$
   c. $f(n+1) = f(n)^2 + f(n-1)^3$
   d. $f(n+1) = f(n) / f(n-1)$

5. (5%) Suppose that there are 9 faculty members in the mathematics department and 11 in the computer science department. How many ways are there to select a committee to develop a discrete mathematics course at a school if the committee is to consist of three faculty members from the mathematics department and four from the computer science department?

6. (10%) What is the coefficient of $x^{12}y^{13}$ in the expansion of $(x+y)^{25}$?

7. (10%) How many solutions does the equation $x_1 + x_2 + x_3 = 11$ have, where x_1, x_2, x_3 are nonnegative integers?

8. (10%) How many different strings can be made by reordering the letters of the word SUCCESS?

9. (10%) What is the expected value of the sum of the numbers that appear when a pair of fair dice is rolled?

10. (10%) Show by induction that if n is a positive integer, then $1 + 2 + \ldots + n = n(n+1)/2$