

淡江大學 97 學年度進修學士班轉學生招生考試試題

13-

系別：資訊工程學系三年級

科目：離散數學

可否使用計算機			
可		否	

本試題共 7 大題， 1 頁

1. A natural number (自然數) greater than 1 and not exceeding 100 must be prime (質數) or divisible (整除) by 2, 3, 5 or 7.
 - (a) Find the number primes not exceeding 100. (10%)
 - (b) Find the number of natural numbers not exceeding 100 and which are either prime or even. (10%)

2. For each of the following relation \mathfrak{R} on \mathbb{Z} , set of integers, determine whether the relation is reflexive, symmetric or transitive. (15%)
 - (a) $(a, b) \in \mathfrak{R}$ if $a \leq b$
 - (b) $(a, b) \in \mathfrak{R}$ if $a > b$
 - (c) $(a, b) \in \mathfrak{R}$ if a divides (整除) b
 - (d) $(a, b) \in \mathfrak{R}$ if $a + b$ is odd (奇數)
 - (e) $(a, b) \in \mathfrak{R}$ if $a^2 = b^2$

3. Determine which of the following are *satisfiable*: (10%)
 - (a) $(P \vee Q \vee R) (\neg P \vee \neg Q \vee \neg R) (\neg P \vee Q) (\neg Q \vee R)$
 - (b) $(P \wedge Q) \vee R \Rightarrow (\neg Q \vee \neg R)$
 - (c) $\neg(P \wedge Q) \Rightarrow \neg P \vee Q$
 - (d) $\neg(P \vee Q) \Rightarrow \neg P \vee Q$
 - (e) $P \wedge Q \Rightarrow \neg Q$

4. Prove or disprove the following are *tautologies*:
 - (a) $((P \wedge Q) \Rightarrow R) \Leftrightarrow ((P \wedge \neg R) \Rightarrow \neg Q)$ (5%)
 - (b) $\neg(P \wedge Q) \Leftrightarrow \neg P \vee Q$ (5%)

5. Determine the coefficients of terms
 - (a) $x^3 y^2$ in $(2x + 3y)^5$ (10%)
 - (b) $x^2 y^3$ in $(2x - 3y + 2)^7$ (10%)

6. Find the value of k such that the inequality $n^3 < 2^n$ always holds (成立) for all $n \geq k$. Prove your answer using mathematical induction. (15%)

7. Solve the following recurrence relation using generating function method.

$$a_n - a_{n-1} - a_{n-2} = 0 \quad n \geq 2, \quad a_0 = 0, \quad a_1 = 1 \quad (10\%)$$