淡江大學九十三學年度進修學士班轉學生招生考試試題 17-1

系別：資訊工程學系三年級　　科目：離 散 數 學

准帶項目請打「○」否則打「×」
簡單型計算機

節次：7 月 14 日第 4 節

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1. Determine the sets A, B, C, and D if
   (a) $A - B = \{1, 2, 7, 11\}$, $B - A = \{3, 6, 9\}$, and $A \cap B = \{4, 8\}$ (5%)
   (b) $C - D = \{1, 2, 5\}$, $D - C = \{7, 9\}$, and $C \cup D = \{1, 2, 4, 5, 6, 7, 8, 9\}$ (5%)

2. With n a positive integer, evaluate the sum
$$\binom{n}{0} + \binom{n}{1}\cdot 2 + \binom{n}{2}\cdot 2^2 + \ldots + \binom{n}{k}2^k + \ldots + \binom{n}{n}\cdot 2^n \quad (10\%)$$

3. Construct a truth table for each of the following compound statements, where p, q, r are primitive statements, and then determine whether they are tautologies or not.
   (a) $\neg(p \vee \neg q) \rightarrow \neg p$ (5%)
   (b) $(p \wedge q) \rightarrow p$ (5%)

4. Each of the following functions $f: \mathbf{Z} \rightarrow \mathbf{Z}$ ($\mathbf{Z}$: set of all integers), determine whether the function is one-to-one and whether it is onto. If the function is not onto, determine its range $f(\mathbf{Z})$ (10%)
   (a) $f(x) = x^2$
   (b) $f(x) = x^2 + x$

5. Prove each of the following for all $n \geq 1$ by the principle of Principle of Mathematical Induction
   (a) $1 + 2 + 3 + \ldots + n = n(n+1)/2$ (10%)
   (b) $(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta$ where $i \in \mathbf{C}$ (complex number) and $i^2 = -1$.
   (hint : consider equation $(\cos\theta + i\sin\theta)^2 = \cos 2\theta + i\sin 2\theta$) (10%)

6. Solve the following recurrence relation. (answer should not involve complex numbers.)
$$a_n = 5\,a_{n-1} + 6\,a_{n-2}, \quad n \geq 1, \quad a_0 = 1, a_1 = 3 \quad (10\%)$$

7. Find the coefficient of x^{83} in $f(x) = (x^5 + x^8 + x^{11} + x^{14} + x^{17})^{10}$ (10%)

8. Find the number of permutations of $a, b, c, \ldots, x, y, z$ in which none of the patterns *spin*, *game*, *path*, or *net* occurs. (hint: use Principle of Inclusion and Exclusion) (10 %)

9. Let $a, b \in \mathbf{Z}^+$. Prove or disprove that if $a^2 \mid b^2$ then $a \mid b$. (10%)