

1. Find the total number of integer solutions that satisfy
(20%) $x_1 + x_2 + x_3 + x_4 = 19$, $0 \leq x_i < 8$, $1 \leq i \leq 4$.
2. Solve the following recurrence relations (No final answer should involve complex numbers)
- (15%) (a) $9a_{n+2} + 12a_{n+1} + 4a_n = 0$, $n \geq 0$, $a_0 = 1$, $a_1 = 4$
- (15%) (b) $a_{n+2} + 4a_n = 0$, $n \geq 0$, $a_0 = 0$, $a_1 = 3$
- (3) Find the Hamming Codes for the following.
- (15%) (a) 1011, 0110
- (15%) (b) 011010, 1100
- (4) Use generating functions to determine how many
(20%) four-element subset of $S = \{1, 2, 3, \dots, 14, 15\}$ contains no consecutive integers.