系別：統計學系二年級　　　　科目：微　積　分

准帶項目請打「○」否則打「×」

| × | 簡單型計算機 |

本試題共 **2** 頁

1. Let $f(x,y) = y^2 - \sin(x)$.

   (a) Find all the critical points of $f(x,y)$.　　　**(6 points)**

   (b) Using the D-Test to determine whether the above points is a relative maximum, a relative minmum, or a saddle point.　　　**(8 points)**

2. (a) State the Chain Rule.

   (b) Use the Chain Rule to find $\dfrac{d}{dx}\log_2(3x^2 - 1)$.

   　　　(10 points)

3. Find X' for $X = X(t)$ in the form of
   $$X^2 - t^2 X + t^3 + 11 = 0.$$　　　**(8 points)**

4. Let $f(x) = \ln(x)$, find a 4-th degree polynomial of x, say $p(x)$, such that $f(1) = p(1)$, and $\dfrac{d^n}{dx^n}f(1) = \dfrac{d^n}{dx^n}p(1)$ for $n = 1,2,3$, and 4.　　　**(8 points)**

5. Evaluate the following integrals.　　　**(18 points)**

   (a) $\displaystyle\int \frac{3e^x}{e^x + e^{-x}}\,dx$　　　(b) $\displaystyle\int_0^\infty x e^{-x}\,dx$

   (c) $\displaystyle\int \tan^2 x\,dx$

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6. Evaluate the double integral $\int_0^1 \int_1^2 \dfrac{x}{\sqrt{x^2 + y}}\, dxdy$.

   (10 points)

7. Find the following limits. (16 points)

   (a) $\lim\limits_{x \to \infty} \dfrac{e^{-x}}{\sin(1/x)}$ 　　　　(b) $\lim\limits_{x \to \infty} \dfrac{\ln(x^x)}{2^x}$

8. Find the value of x between 1 and 10 such that minimizes the value of the function defined by

   $\int_1^{(x^2+1)} (\mu^2 - 1)^{1/2}\, d\mu$ (8 points)

9. Find the radius of convergence of the power series:

   $\sum\limits_{n=0}^{\infty} \dfrac{n^3(x - 3)^n}{3^n}$ (8 points)