淡江大學 102 學年度進修學士班轉學生招生考試試題

系別：統計學系三年級　　科目：機率與管理數學

考試日期：7 月 22 日(星期一) 第 3 節　　本試題共 7 大題， 1 頁

1. Evaluate the integrals (1) $\int_{-1}^{1}\int_{0}^{2}\left(1-6x^2y\right)dxdy$, (2) $\int \ln x\, dx$. (16%)

2. Let $f(x,y)=x\cos y+ye^x$, find $\dfrac{\partial^2 f}{\partial x\partial y}$. (8%)

3. Find the Taylor series generated by $f(x)=\cos x$ at $x=0$. (10%)

4. The random variable X has cumulative distribution function F given by:

$$F(x)=\begin{cases}0, & x\le 0\\ 2c(x^2-\frac{1}{3}x^3), & 0<x\le 2\\ 1, & x>2\end{cases}.$$

（1）Determine the corresponding probability density function and the constatnt c. (12%)

（2）Calculate the probability $P(X>\frac{1}{2})$. (8%)

5. Let the joint probability density function of X and Y be $f_{X,Y}(x,y)=\dfrac{21}{2}x^2y$ for $0<x^2<y<1$ and $x>0$.

（1）Determine the marginal probability density function $f_Y(\cdot)$. (10%)

（2）Determine the conditional probability density function $f_{X|Y}(\cdot\,|\,y)$. (8%)

（3）Calculate $E(X\,|\,Y=y)$ for given $0<y<1$. (8%)

6. Let the random variable X have the Exponential distribution with parameter λ and set $Y=\log X$. Determine the probability density function $f_Y(\cdot)$. (10%)

7. Let $\{A_1,A_2,A_3,A_4,A_5\}$ be a partition of the sample space S and suppose that: $P(A_j)=\dfrac{j}{15}$ and $P(A|A_j)=\dfrac{5-j}{15}$, $j=1,\cdots,5$. Compute the probability $P(A_1|A)$. (10%)