淡江大學九十二學年度進修學士班轉學生招生考試試題

系別：統計學系三年級　　科目：機率與管理數學

准帶項目請打「○」否則打「×」	
×	簡單型計算機

本試題雙面印製

1) (a) State the Bayes' Formula. (8%)

(b) SARS, in Taiwan, is present in $100p_1\%\ (0<p_1<1)$ of the population. A diagnostic test is available but is not perfect yet. The test shows positive with probability $p_2\ (0<p_2<1)$ and negative with probability $1-p_2$. For a SARS patient, the test shows negative with probability $p_3\ (0<p_3<1)$ and positive with probability $1-p_3$. A person is chosen at random from the population. Let D be the event that the person is infected with SARS disease and let E be the event that the person is not infected. Under the diagnostic test;

(i) Determine the probability of the event A that the person tests positive. (6%)

(ii) Given that the test shows positive, what is the probability that the person actually has SARS disease? (6%)

(Note: I want to see the formulas you used in doing these two problems)

2) Let X be a random variable with probability density function (p.d.f.) given by

$$f(x)=\begin{cases} ce^{-2x}, & x>0 \\ -cx, & -1<x\le 0 \\ 0, & x\le -1 \end{cases}$$

(a) Find the constant c. (5%)

(b) Find the mean μ of X. (5%)

(c) Find the distribution function F of X. (7%)

3) Let X,Y be two random variables with joint p.d.f. given by

$$f_{X,Y}(x,y)=\begin{cases} \frac{1}{2}, & 0<x<y<2 \\ 0, & otherwise \end{cases}$$

(a) Find the conditional probability $P(-1<Y<2\,|\,X=1)$. (8%)

(b) Find the conditional expectation $E(Y\,|\,X=1)$ (6%)

4) Let X,Y be two random variables with joint p.d.f. given by

$$f_{X,Y}(x,y)=\begin{cases} \frac{x+y}{21}, x=1,2,3,\ y=1,2 \\ 0, \quad otherwise \end{cases}$$

(a) Find $P(-1<X<2\,|\,Y=1)$. (6%)

(b) Find $E(XY)$. (6%)

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5) Find the following limits: (10%)

(a) $\lim_{x\to 2} \frac{x^2-4}{x^2-5x+6}$ (b) $\lim_{x\to\infty} \left(1+\frac{1}{x^2}\right)^x$

6) Find the following integrals:

(Note: You may use some special functions to do them)

(a) $\int_0^\infty x^2 e^{-x}\,dx$ (5%) (b) $\int_0^1 x^2(1-x)^3\,dx$ (5%)

(c) $\int_1^3 \int_0^\infty xye^{-x}\,dx\,dy$ (6%)

7) (a) State the Taylor's theorem in Calculus. (6%)

(b) Let $f(x)=\frac{1}{e^x}$. Find the Taylor series of f represents f at $x=0$. (5%)