10

淡江大學 103 學年度日間部轉學生招生考試試題

系別：數學學系二年級　　　　科目：線性代數

考試日期：7月19日(星期六) 第4節　　　　本試題共 6 大題， 1 頁

1. (10 pts) Determine if the set $S = \{(1,2,3),(0,1,1),(2,5,1)\}$ forms a basis for $\mathbb{R}^3$.

2. (30 pts) Let $A = \begin{bmatrix} 7 & -6 \\ 3 & -2 \end{bmatrix}$.

(a) Find the eigenvalues of A.
(b) Find an invertible matrix P such that $P^{-1}AP$ is diagonal.
(c) Use the result in (b) to compute A^{10}.

3. (20 pts) Let W be the solution space of the following system of linear equations.

$$\begin{aligned} x + y - z + w &= 0 \\ 3x - y + z + w &= 0 \end{aligned}$$

(a) Find an orthonormal basis for W.
(b) What is the dimension of $W^{\perp}$ (orthogonal complement of W)? Justify your answer.

4. (20 pts) Let $M_{2,2}$ be the space of all 2×2 matrices with real entries. Define $T : M_{2,2} \to M_{2,2}$ by

$$T\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = \begin{bmatrix} a+b+c & 2c-d \\ 3b+c+2d & a+b \end{bmatrix}.$$

(a) Find the matrix representation of T relative to the standard basis for $M_{2,2}$:

$$\left\{\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}\right\}.$$

(b) Determine if T is invertible.

5. (10 pts) Let $T : \mathbb{R}^n \to \mathbb{R}^m$ be a linear transformation. If $\{v_1, v_2, \cdots, v_k\}$ is a linearly dependent set in $\mathbb{R}^n$, show that the set $\{T(v_1), T(v_2), \cdots, T(v_k)\}$ is also linearly dependent.

6. (10 pts) Suppose matrices A and B have the same characteristic polynomial

$$|\lambda I - A| = |\lambda I - B| = (\lambda - 1)(\lambda - 2)(\lambda - 3)(\lambda - 4).$$

Is it true that A and B must be similar? Why?