淡江大學 101 學年度轉學生招生考試試題

系別：數學學系二年級　　科目：線性代數

考試日期:7月16日(星期一) 第4節　　本試題共 6 大題， 1 頁

1. (13%) Find the matrix representation of $T(x_1, x_2, x_3) = (2x_1 + 3x_2 + x_3, 3x_1 + 3x_2 + x_3, 2x_1 + 4x_2 + x_3)$ in standard basis, and then find the inverse of T.

2. (12%) Determine whether the followings are linearly dependent or linearly independent.

   (a) $p_1 = 1 - x, p_2 = 5 + 3x - 2x^2, p_3 = 1 + 3x - x^2$.

   (b) $v_1 = (1,-2,3), v_2 = (5,6,-1), v_3 = (3,2,1)$.

3. (25%) Let $A = \begin{bmatrix} 1 & -1 & -1 \\ 1 & 3 & 1 \\ -3 & 1 & -1 \end{bmatrix}$.

   (a) Find the determinant of A.
   (b) Find the characteristic polynomial, eigenvalues, and corresponding eigenvectors of A.
   (c) Find an invertible matrix P and diagonal matrix D such that $D = P^{-1}AP$.

4. (20%) Let W be spanned by $v_1 = (1,1,1,-1), v_2 = (2,-1,-1,1), v_3 = (-1,2,2,1)$. Consider the standard inner product on $\mathbb{R}^4$.
   (a) Find an orthonormal basis for W.
   (b) Find the orthogonal projection of the vector $(1,0,0,1)$ on W.

5. (15%) Define $T : \mathbb{R}^3 \to \mathbb{R}^2$ as $T(x, y, z) = (x - y, 2z)$.
   (a) Find the range and the null space of T.
   (b) Find the nullity and rank of T.

6. (15%) Let $T : \mathbb{R}^n \to \mathbb{R}^m$ be a linear 1-1 transformation. Suppose $\{x_1, \ldots, x_k\}$ is linearly independence set in $\mathbb{R}^n$, prove that $\{Tx_1, \ldots, Tx_k\}$ are linearly independent.