淡江大學 100 學年度轉學生招生考試試題

系別：數學學系二年級　　　　科目：線性代數

考試日期：7月18日(星期一) 第4節　　　本試題共 6 大題， 2 頁

Show your work

1. Decide whether each of the following sets of vectors is linearly dependent or linearly independent. (20%)

(a) $\left\{ \begin{bmatrix} 1 \\ -1 \\ 3 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ -2 \end{bmatrix}, \begin{bmatrix} 3 \\ -1 \\ 4 \end{bmatrix} \right\}$

(b) $\left\{ \begin{bmatrix} -1 \\ -1 \\ 2 \\ -2 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ 3 \\ -1 \\ 4 \end{bmatrix} \right\}$

(c) $\left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} \right\}$

2. Let $A, B,$ and C denote $n \times n$ matrices and assume that $|A| = 2$, $|B| = -3$, $|C| = 4$. Evaluate $|BA^2C^{-1}|$.

($|A|$ denote the determinant of A) (10%)

3. Let $A = \begin{bmatrix} 2 & 7 & 1 \\ 1 & 4 & -1 \\ 1 & 1 & 0 \end{bmatrix}$, (10%)

(a) Find A^{-1}.

4. Let $T : R^m \to R^n$ be a linear transformation, find A such that $T(\vec{X}) = A\vec{X}$, $\vec{X} \in R^m$ for each of the following: (20%)

(a) $T : R^3 \to R^4$, $T\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} 2x - 3z \\ 4y + 5z \\ x - 2y + 7z \\ 4z \end{bmatrix}$.

(b) $T : R^2 \to R^2$, $T\left(\begin{bmatrix} 1 \\ 2 \end{bmatrix}\right) = \begin{bmatrix} 6 \\ -1 \end{bmatrix}$, $T\left(\begin{bmatrix} 3 \\ 4 \end{bmatrix}\right) = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$.

(c) $T : R^2 \to R^2$, T is the composition of the rotation through $\pi/2$ followed by the reflection in the line y = x.

(d) $T : R^2 \to R^2$, T is the reflection in the line y = 5x.

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5. Let A be a diagonalizable nxn matrix with eigenvalues $\lambda_1, \lambda_2, \cdots, \lambda_n$ (including multiplicities), show that $|A| = \lambda_1 \lambda_2 \cdots \lambda_n$ and tr(A) = $\lambda_1 + \lambda_2 + \cdots + \lambda_n$. (20%)

6. Let $T: R^6 \to R^4$ be a linear transformation defined by $T(\vec{X}) = A\vec{X}$, $\vec{X} \in R^6$, where

$$A = \begin{pmatrix} 1 & -2 & 3 & 0 & 0 & 2 \\ 2 & -5 & 6 & -2 & -3 & 4 \\ 0 & 5 & 0 & 10 & 15 & 0 \\ 2 & 0 & 6 & 8 & 18 & 4 \end{pmatrix}$$

. Find bases for ker(T) and Ran(T). (20%)