淡江大學 99 學年度轉學生招生考試試題

系別：數學學系二年級　　　　　　科目：線　性　代　數

本試題共 六 大題， 一 頁

Partial credit—You must show all your work.

1. (20 %) Find the characteristic polynomial and the eigenvalues for the 3×3 matrix $A = \begin{bmatrix} 3 & 1 & -2 \\ 12 & 0 & -10 \\ 2 & 1 & -1 \end{bmatrix}$.

2. (10 %) Find the inverse matrix B^{-1} for the 3×3 matrix $B = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 1 & 3 & 1 \end{bmatrix}$.

3. (20 points) Let $A = \begin{bmatrix} 3 & -1 & -1 \\ -12 & 0 & 5 \\ 4 & -2 & -1 \end{bmatrix}$.

(1) Find an invertible 3×3 matrix P and a 3×3 diagonal matrix D such that $P^{-1}AP = D$.

(2) Calculate A^5.

4. (20 points) Let

$$W = \left\{ \mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} \middle| \, x_1 = -2x_2 + x_4, \ x_3 = -x_4 \right\}$$

(1) Find a basis B for W and then find $\dim(W)$

(2) Applying Gram-Schmidt Orthogonalization on B, find an orthogonal basis $\widetilde{B}$ for W

5. (20 points) Let V be the vector space of 2×2 matrices, and let $T : V \to V$ be defined by

$$T\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = \begin{bmatrix} -3a + 5d & 3b - 5c \\ -2c & 2d \end{bmatrix}.$$

Find the matrix of T with respect to the basis $C = \{A_1, A_2, A_3, A_4\}$, where

$$A_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \quad A_3 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \quad A_4 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}.$$

6. (10 points) Let $T : U \to V$ be a linear transformation and let U be finite dimensional. Show that if $\dim(U) < \dim(V)$, then T is not onto.