淡江大學98學年度轉學生招生考試試題

系別：數學學系二年級　　　　科目：線　性　代　數

准帶項目請打「V」	
	計算機

本試題共 8 大題，　頁

1. (10%) Find the inverse of $\begin{bmatrix} 1 & 4 & -1 \\ 2 & 7 & 1 \\ 1 & 3 & 0 \end{bmatrix}$

2. (10%) Given that D is diagonal and nonsingular and that $D=(I+A)^{-1}A$ prove that A is diagonal also.

3. (10%) Let A be 3×3 matrix, and let $v_1, v_2, \ldots, v_n$ be linearly independent vectors in R^n expressed as $n\times 1$ matrices. What must be true about A for $Av_1, Av_2, \ldots, Av_n$ to be linearly independent?

4. (10%) Prove the Cauchy- Schwarz Inequality
   If u and v are vectors in a real inner product space, then
   $$|\langle u, v\rangle| \le \|u\|\|v\|$$

5. (20%) Consider the vector space R^3 with the Euclidean inner product.
   (1) Apply the Gram-Schmidt process to transform the basis vectors $u_1=(1,1,1)$, $u_2=(0,1,1)$, $u_3=(0,0,1)$ into an orthonormal basis $\{q_1, q_2, q_3\}$.
   (2) Find the QR-decomposition of the matrix $A=\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$.

6. (20%) Let $T: R^3 \to R^3$ be a linear operator given by $T\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right)=\begin{bmatrix} -2x_3 \\ x_1+2x_2+x_3 \\ x_1+3x_3 \end{bmatrix}$.
   Let $B=\{(1,0,0),(0,10),(0,0,1)\}$ be a basis for R^3.
   (1) Find $[T]_B$ (the matrix of T with respect to the basis B).
   (2) Find a basis B_1 for R^3 such that $[T]_{B_1}$ is diagonal.

7. (10%) Find a matrix P that diagonalizes $A=\begin{bmatrix} 1 & 0 & 0 \\ 1 & 2 & 0 \\ -3 & 5 & 2 \end{bmatrix}$

8. (10%) Suppose $T: V \to W$ is a linear transformation from n-dimension vector space V to a vector space W and $\{v_1, v_2, \ldots, v_n\}$ is a basis of V. Show that if $\{v_1, v_2, \ldots, v_r\}$, where $1 \le r < n$, is a basis of ker T then $\{T(v_{r+1}), T(v_{r+2}), \ldots, T(v_n)\}$ is a basis of rank T.